Ain Shams Engineering Journal (2015) 6, 661–669

Ain Shams University

Ain Shams Engineering Journal

www.elsevier.com/locate/asej
www.sciencedirect.com

ENGINEERING PHYSICS AND MATHEMATICS

Double diffusive Darcy flow induced by a spherical source

R. Ganapathy [a,*], A. Mohan [b,1]

[a] Faculty of Sciences, MAM College of Engineering, Anna University, Tiruchirappalli 621105, India
[b] Faculty of Sciences, Saranathan College of Engineering, Anna University, Tiruchirappalli, 620012, India

Received 14 May 2014; revised 6 November 2014; accepted 26 November 2014
Available online 9 February 2015

KEYWORDS

Spherical source;
Porous medium;
Darcy flow;
Double diffusion;
Rayleigh number

Abstract A simple mathematical theory is proposed for the study of the free convective mass transfer flow induced by a spherical source in an unbounded porous medium assuming the validity of the Darcy flow model. Besides generating heat, the source generates a chemical substance too at a constant rate. Assuming the heat generation rate not excessive, an exact analytical solution is obtained for the flow field using the method of superposition for the determination of the temperature and concentration fields. The significance of the impact of the species concentration gradients upon the thermally driven flow has been highlighted. The results are delineated by comparing them with those of a point source and the evolution of the flow field is contrasted with that due to the presence of a heated sphere. Solutions for sources with spherical geometry could be deduced algebraically from the results of this study.

1. Introduction

Among the thermal engineering applications which benefit from a better understanding of the fundamentals of heat and fluid flow in a porous medium are thermal insulations, geothermal systems, cooling of nuclear reactors and underground disposal of nuclear waste. In particular, those concerning subsea-bed disposal of nuclear waste attract greater attention in view of their growing relevance to the successful containment of the transport of radio-nuclide into the water columns and several results are available in the literature modeling the situation with heat sources buried in unbounded porous media. In most of the existing published studies [1–7], the source is assumed to be concentrated at one point although in practice, the nuclear waste is first encapsulated in a suitably designed container before being implanted into the sedimentary layer below the subsea-bed [4,5]. Hence, to have a better understanding of the complexity of the situation, a different modeling, other than that of a concentrated point source, may be needed in order to have a comprehensive estimate of the overall convection effects, the most appropriate one with spherical geometry being that of a spherical source, which indeed is the focus of our investigation in the present study. Motivated by the significance of understanding such free convection flows, Hardee [8]

* Corresponding author. Mobile: +91 97894 99494.
E-mail addresses: sowgar05@yahoo.com (R. Ganapathy), appavumohan@gmail.com (A. Mohan).
[1] Tel.: +91 9894618086.
Peer review under responsibility of Ain Shams University.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.asej.2014.11.009
2090-4479 © 2014 Faculty of Engineering, Ain Shams University. Production and hosting by Elsevier B.V.

investigated the phenomenon of buoyancy-driven thermal convection due to the presence of a spherical source in an unbounded porous medium using a boundary-layer approximation while, Hodgkinson [9] investigated the same associated with an idealized waste depository in a permeable hard rock mass. With the exception of these two works no other work seems to be available in the literature on such free convection flows due to the presence of spherical sources in porous media. Of course, a few other papers are available in the literature using spherical geometry [10–17], but they essentially deal with heated spheres only. In this context, the work of Ganapathy [18] deserves mention.

In most practical situations, quite often, species concentration gradients greatly affect the flow and as a result they play a decisive role in the development of the flow field [19]. Even though the importance of this class of problems has been to some extent established in the literature [20–24], yet, those concerning double diffusion due to spherical sources have remained largely overlooked which indeed has motivated our interest in the present topic. The analysis of such flows is essential for the solution of many engineering problems such as the spreading of pollutants created by an exothermic reaction at an underground site. Besides its importance in geophysics, the problem finds its applications in chemical engineering too. Although the broader problem of double diffusion depends on two Rayleigh numbers, one based on the thermal field and the other on the concentration field, yet for simplicity, we formulate the problem in such a way that the impact of the species concentration gradients upon the thermally driven flow is measured by a parameter N [19]. It may be noted that all discussions in this work focus on the case where the net flow is upward.

It may be mentioned here that Jaballah et al. [25], by considering the mixed convection in a channel with porous layers and using a thermal non-equilibrium model have shown that in respect of the thermal equilibrium model, the quantity of heat is preserved within the channel for a fluid injected with at velocity well determined and a heat flow imposed, so that the quantity of heat collected by the solid particles is yielded to the fluid particles without loss which is in conformity with our assumption in the present work.

In conclusion, the main goal of this article is to investigate analytically the effect of species diffusion on the buoyancy induced free convective mass transfer flow from a spherical source in an unbounded porous medium assuming the validity of the Darcy flow model. Assuming the heat generation rate not excessive, an exact analytical solution is obtained for the flow field using the method of superposition for the determination of the temperature and concentration fields around the spherical source. As a summary of what is presented below, the mathematical problem is formulated in Section 2 and the method of solution is presented in Section 3, with a discussion on the flow field in Section 4. Finally, we conclude the study with a review of the results obtained.

2. Mathematical formulation

We consider the free convective mass transfer flow around a continuous spherical source of radius a, buried in an unbounded porous medium of low permeability, from which a quantity $\rho c Q$ of heat is liberated together with a substance at the rate m [kg s^{-1}], where ρ is the fluid density, c the specific heat at constant pressure and Q, the thermal energy of the source. The medium is assumed to be rigid, homogeneous and isotropic and the fluid saturating the medium Boussinesq incompressible with the density–temperature–concentration relation:

$$\rho = \rho_\infty\{1 - (\beta T + \beta_c C)\}, \tag{1}$$

where (β, β_c) are respectively the volumetric coefficients of thermal and concentration expansion, (T, C) are the temperature and species concentration in excess of the ambient and the subscript ∞ denoting a reference state. An illustration of the physical setup with the spherical source embedded in the unbounded porous medium together with the configuration of interest is given in Fig. 1. A spherical-polar co-ordinate system (r, φ, θ) is chosen with the origin at the center of the spherical source and the axis $\varphi = 0$ vertically upward and parallel but opposite to the gravity vector. Consequently, as the problem is symmetrical in the angular direction θ, neither θ nor the θ-component of velocity appears in the analysis. Thus, assuming the Darcy flow model, we have for the conservation of mass, momentum, energy and species concentration in the medium in the absence of dispersion [26]:

$$\partial(r^2 u \sin\varphi)/\partial r + \partial(r v \sin\varphi)/\partial\varphi = 0, \tag{2}$$

$$u = -(K/\mu)(\partial P/\partial r + \rho g \cos\varphi), \tag{3}$$

$$v = -(K/\mu)(r^{-1}\partial P/\partial\varphi - \rho g \sin\varphi), \tag{4}$$

$$\sigma\partial T/\partial t + u\partial T/\partial r + (v/r)\partial T/\partial\varphi = \alpha\Delta T, \tag{5}$$

$$\varepsilon\partial C/\partial t + u\partial C/\partial r + (v/r)\partial C/\partial\varphi = D\Delta C, \tag{6}$$

where (u, v) are the radial and transverse components of the mean filtration velocity of the fluid, g the acceleration due to gravity, t the time, K the medium permeability, μ the coefficient of viscosity, P the fluid pressure in excess of its hydrostatic value, ε the porosity of the porous matrix, α the effective thermal diffusivity of the medium which is equal to the ratio of thermal conductivity k of the porous medium filled with the stagnant fluid divided by the heat capacity $(\rho c)_f$ of the

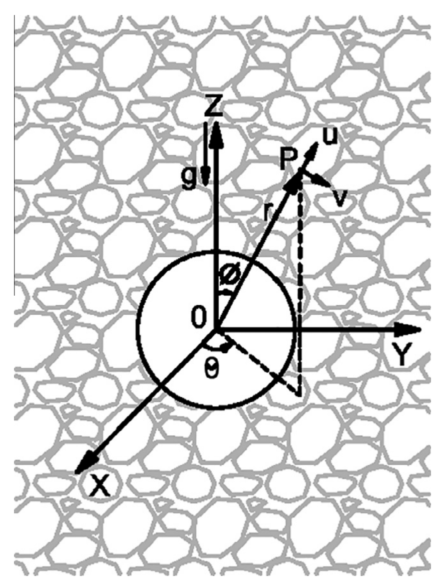

Figure 1 Physical setup of the spherical source embedded in an unbounded porous medium. Configuration of interest: Spherical-polar co-ordinate system (r, φ, θ) with the origin at the center of the spherical source.