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An exponential expansion method and its application to the strain wave equation in microstructured solids

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Abstract The modeling of wave propagation in microstructured materials should be able to account for various scales of microstructure. Based on the proposed new exponential expansion method, we obtained the multiple explicit and exact traveling wave solutions of the strain wave equation for describing different types of wave propagation in microstructured solids. The solutions obtained in this paper include the solitary wave solutions of topological kink, singular kink, non-topological bell type solutions, solitons, compacton, cuspon, periodic solutions, and solitary wave solutions of rational functions. It is shown that the new exponential method, with the help of symbolic computation, provides an effective and straightforward mathematical tool for solving nonlinear evolution equations arising in mathematical physics and engineering.

1. Introduction

Nonlinear evolution equations (NLEEs) are very important model equations in mathematical physics and engineering for describing diverse types of physical mechanisms of natural phenomena in the field of applied sciences and engineering. For this reason, the search of exact traveling wave solutions to NLEEs plays very important role in the study of these physical phenomena. The wave propagation phenomena are observed in microstructured solids, plasma physics, chemical physics, elastic media, optical fibers, fluid dynamics, quantum mechanics, etc. With the rapid development of nonlinear science based on computer algebraic system, many effective methods have been presented, such as, the tanh method [1], the extended tanh method [2,3], the modified extended tanh-function method [4,5], the Exp-function method [6–8], the improved F-expansion method [9], the $\exp(-\Phi(\xi))$-expansion method [10–14], the sine–cosine method [15], the modified simple equation method [16–22], the (G'/G)-expansion method [23,24], the novel (G'/G)-expansion method [25], new approach of the generalized (G'/G)-expansion method [26,27], the Jacobi elliptic function method [28,29], the homogeneous balance method [30–32], the Hirota's bilinear method [33], the homotopy perturbation technique [34] and others.

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The mathematical modeling of wave propagation in microstructured solids is also described by nonlinear PDEs and should be able to account for several scales of microstructure. If the scale reliance involves dispersive effects and materials behave nonlinearity, then dispersive and nonlinear effects may be balanced and thus the solitary wave causes. The existence and appearance of solitary waves in complicated physical problems apart from the model equations of mathematical physics must be analyzed with sufficient accuracy. There is an amount of paper where the governing equations for waves in microstructured solids have been derived and the solitary waves were analyzed [27,35–38]. For instance, the microstrain wave function $u(x, t)$ in micro-structured solids is characterized by nonlinear partial differential equation. The governing nonlinear PDE equation (see Ref. [27,37]) for the microstrain wave function $u(x, t)$ in micro-structured solids is given by

$$u_{tt} - u_{xx} - \varepsilon\alpha_1(u^2)_{xx} - \gamma\alpha_2 u_{xxt} + \delta\alpha_3 u_{xxxx} - (\delta\alpha_4 - \gamma^2\alpha_7)u_{xxtt} + \gamma\delta(\alpha_5 u_{xxxxt} + \alpha_6 u_{xxttt}) = 0. \quad (1)$$

where ε accounts for elastic strains, δ characterizes the ratio between the microstructure size and the wave length, γ characterizes the influence of dissipation and $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ are constants.

The balance between nonlinearity and dispersion takes place when $\delta = O(\varepsilon)$. If we set $\gamma = 0$, we have the non-dissipative case, and governed by the double dispersive equation [27,37] as follows:

$$u_{tt} - u_{xx} - \varepsilon\{\alpha_1(u^2)_{xx} - \alpha_3 u_{xxxx} + \alpha_4 u_{xxtt}\} = 0. \quad (2)$$

The aim of this paper was to apply the proposed exponential expansion method to construct the new exact traveling wave solutions to the strain wave Eq. (2) for describing various types of solitary wave propagation in microstructured solids.

The remainder of the paper is organized as follows: In Section 2, we give the brief description of the proposed new exponential expansion method. In Section 3, we apply this method for finding the explicit and solitary wave solutions to the strain wave equation in microstructured solids. The physical explanations of the obtained solutions and the advantages of the new exponential expansion method are presented in Sections 4 and 5 respectively. Conclusions are given in the last section.

2. Description of the proposed exponential expansion method

This section presents the brief descriptions of the new exponential expansion method.

Let us consider the NLEE as follows:

$$F(u, u_t, u_x, u_{xx}, u_{tt}, u_{tx}, \ldots) = 0, \quad (3)$$

where F is a function of $u, u_t, u_x, u_{xx}, u_{tt}, u_{tx}, \ldots$ and the subscripts denote the partial derivatives of $u(x, t)$ with respect to x and t.

Suppose $u(x, t) = u(\xi),\ \xi = x \pm Vt$, where the constant V is the velocity of the traveling wave, then the Eq. (3) reduces to a nonlinear ordinary differential equation (ODE) for $u = u(\xi)$:

$$Q(u, u', u'', u''', \cdots) = 0, \quad (4)$$

where Q is a function of $u, u', u'', u''', \cdots$ and its derivatives point out the ordinary derivatives with respect to ξ.

Let us consider the traveling wave solution of Eq. (4) is of the form:

$$u(\xi) = \sum_{i=0}^{N} A_i(\exp(-\Phi(\xi)))^i,\ A_N \neq 0 \quad (5)$$

where the coefficients $A_i(0 \leqslant i \leqslant N)$ are constants to be evaluated and $\Phi = \Phi(\xi)$ satisfies the first order nonlinear ordinary differential equation:

$$\Phi'(\xi) = \exp(-\Phi(\xi)) + \mu\exp(\Phi(\xi)) + \lambda, \quad (6)$$

where λ and μ are arbitrary constants.

The value of the positive integer N can be determined by balancing the highest order derivatives with the nonlinear terms of the highest order appearing in Eq. (4).

By substituting (5) into (4) and using (6) when required, we obtain a system of algebraic equations for $A_i(0 \le i \le N)$, λ, μ, and V. With the help of symbolic computation, such as, Maple, we can evaluate the obtaining system and find out the values $A_i(0 \le i \le N)$, λ, μ, and V. It is notable that Eq. (6) has the following five types of general solutions [10–14]:

$$\Phi(\xi) = \ln\left(\frac{-\sqrt{\Theta}\tanh\{0.5\sqrt{\Theta}(\xi+\xi_0)\} - \lambda}{2\mu}\right), \mu \neq 0, \Theta = \lambda^2 - 4\mu > 0, \quad (7a)$$

$$\Phi(\xi) = \ln\left(\frac{\sqrt{-\Theta}\tan\{0.5\sqrt{-\Theta}(\xi+\xi_0)\} - \lambda}{2\mu}\right), \mu \neq 0, \Theta = \lambda^2 - 4\mu < 0, \quad (7b)$$

$$\Phi(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi+\xi_0)) - 1}\right), \mu = 0, \lambda \neq 0, \Theta = \lambda^2 - 4\mu > 0, \quad (7c)$$

$$\Phi(\xi) = \ln\left(-\frac{2(\lambda(\xi+\xi_0)+2)}{\lambda^2(\xi+\xi_0)}\right), \mu \neq 0, \lambda \neq 0, \Theta = \lambda^2 - 4\mu = 0, \quad (7d)$$

$$\Phi(\xi) = \ln(\xi + \xi_0),\ \lambda = 0,\ \mu = 0. \quad (7e)$$

where ξ_0 is the integration constant.

Thus the multiple explicit solutions to the NLEE (3) are obtained by means of the Eqs. (5) and (7).

Again, suppose Eq. (4) has solution of the form (5) and $\Phi = \Phi(\xi)$ satisfies another first order nonlinear ordinary differential equation:

$$\Phi'(\xi) = -\sqrt{\lambda + \mu(\exp(-\Phi(\xi)))^2},\ \lambda, \mu \in \Re. \quad (8)$$

By substituting (5) into (4) and using (8) repeatedly, we obtain a system of algebraic equations for $A_i(0 \le i \le N)$, λ, μ, V. With the help of symbolic computation, such as, Maple we can evaluate the resulting system and find out the values $A_i(0 \le i \le N)$, λ, μ, V. It is notable that Eq. (8) has the following general solutions:

$$\Phi(\xi) = -\ln\left(-\sqrt{\frac{\lambda}{\mu}}\operatorname{csc}h\left[\sqrt{\lambda}(\xi+\xi_0)\right]\right), \lambda > 0,\ \mu > 0, \quad (9a)$$

$$\Phi(\xi) = -\ln\left(\sqrt{\frac{-\lambda}{\mu}}\sec\left[\sqrt{-\lambda}(\xi+\xi_0)\right]\right), \lambda < 0, \mu > 0, \quad (9b)$$

$$\Phi(\xi) = -\ln\left(\sqrt{\frac{\lambda}{-\mu}}\sec h\left[\sqrt{\lambda}(\xi+\xi_0)\right]\right), \lambda > 0, \mu < 0, \quad (9c)$$