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Analytical and numerical solution of three-dimensional channel flow in presence of a sinusoidal fluid injection and a chemical reaction

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Slip condition

Abstract Modeling of three-dimensional channel flow in a chemically-reacting fluid between two long vertical parallel flat plates in the presence of a transverse magnetic field is presented. The stationary plate is subjected to a transverse sinusoidal injection velocity distribution while the uniformly moving plate is subjected to a constant suction and slip boundary conditions. Due to this type of injection velocity, the flow becomes three dimensional. Comparisons with previously published work are performed and the results are found to be in excellent agreement. An increase in the permeability/magnetic parameter is found to escalate the velocity near the plate in motion. Growing Reynolds number or magnetic parameter enhances the x-component and reduces the z-component of the skin-friction at the wall at rest. The acquired knowledge in our study can be used by designers to control MHD flow as suitable for certain applications which include laminar magneto-aerodynamics, materials processing and MHD propulsion thermo-fluid dynamics.

1. Introduction

Flows involving the simultaneous diffusion of thermal energy and chemical species have received due attention owing to their importance in geophysical and engineering applications. Geophysical applications include investigation of underground water resources, natural gas and mineral oils. Engineering applications include absorbers, humidifiers and desert coolers.

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Nomenclature

U	uniform motion of the outer plate (m s^{-1})
v_0	injection velocity of the stationary plate (m s^{-1})
$\vec{q}$	velocity vector of the fluid at $(\bar{x}, \bar{y}, \bar{z})$
$\overline{T}_0$	temperature at the stationary plate
$\overline{C}_0$	molar species concentration at the stationary plate
$\overline{T}_1$	temperature at the plate in uniform motion
$\overline{C}_1$	molar species concentration at the plate in uniform motion
$\overline{T}_e$	equilibrium temperature of the fluid
$\overline{C}_e$	equilibrium molar species concentration of the fluid
$\overline{T}$	temperature of the fluid
$\overline{C}$	molar species concentration of the fluid
A	heat sink parameter
g	acceleration due to gravity (m s^{-2})
C_P	specific heat at constant pressure (J kg^{-1} K)
D	chemical molecular diffusivity (m^2 s^{-1})
Q	rate of heat absorption per unit volume per degree Kelvin
$\bar{p}$	fluid pressure (Pa)
Re	Reynolds number
Pr	Prandtl number
Sc	Schmidt number
M	Hartmann number
K	chemical reaction parameter
h	slip parameter
Gr	Grashof number for heat transfer
Gm	Grashof number for mass transfer
m	wall thermal ratio
n	wall mass ratio
B_0	magnetic field strength
d	distance between the plates

Greek symbols

β	coefficient of volume expansion (K^{-1})
$\bar{\beta}$	volumetric coefficient expansion with species concentration (K^{-1})
ε	small reference parameter (ε 1)
θ	dimensionless fluid temperature
ϕ	dimensionless species concentration of the fluid
κ	thermal conductivity (W m^{-1} K^{-1})
ν	kinematic viscosity of the fluid (m^2 s^{-1})
ρ	fluid density (kg m^{-3})
μ	coefficient of viscosity of the fluid
σ	electrical conductivity
τ	shearing stress (N m^{-2})

subscripts

0	conditions at the stationary plate
1	conditions at the uniform motion plate

The principle of controlling the temperature of a heated body and the natural convection on a heated surface by suction of the fluid and heat transfer from the boundary layer to the wall finds its applications in several engineering situations of which a high temperature heat exchanger is one such instance. Mass transfer is used in flow problems in order to evaluate the distribution of species concentration and the corresponding species flux. It is well known that fluid particles may slip at the boundary of the surface, in certain situations of a geothermal region. Further, heat sinks find their use in engineering applications where the prime concern is to dissipate as much heat as possible from a heated surface within a short time.

Suction and injection at the plate also play a fundamental role in the plane Couette flow. It remains two-dimensional if the suction and injection applied at the porous parallel plate are uniform, but by the application of the transverse sinusoidal injection at the stationary plate and constant suction at the moving plate, the flow remains three-dimensional as studied by Singh [1]. A similar problem of three dimensional Couette flow of dusty viscous fluid was investigated by Govindarajan et al. [2] with transpiration cooling. Such flow problems are important for studies of transpiration cooling process by investigating associated heat transfer problems. Zaturska et al. [3] reported on the flow of viscous fluid driven along a channel by suction at porous walls. More recently, King and Cox [4] performed an asymptotic analysis of the steady-state and time-dependent laminar porous channel flows. Das et al. [5] studied the three dimensional Couette flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel porous flat plates in presence of a transverse magnetic field. Sharma and Saini [6] investigated the effect of injection/suction between two horizontal parallel porous flat plates, with transverse sinusoidal injection of fluid at the stationary plate and its corresponding removal by periodic suction through the plate in motion, assuming the sinusoidal injection at the lower plate and its corresponding removal by the upper plate in motion. Makinde and Chinyoka [7] studied the unsteady flow and heat transfer of a dusty fluid between two parallel plates with an external uniform magnetic field is applied perpendicular to the plates with a Navier slip boundary condition. The governing non-linear partial differential equations are solved numerically using a semi-implicit finite difference scheme. A three-dimensional Couette flow through a porous medium with heat transfer has also been investigated by Ahmed [8]. Fasogbon [9] studied the simultaneous buoyancy force effects of thermal and species diffusion through a vertical irregular channel by using parameter perturbation technique. The study of heat and mass transfer on the free convective flow of a viscous incompressible fluid past an infinite vertical porous plate in presence of transverse sinusoidal suction velocity and a constant free stream velocity was presented by Ahmed [10]. Moreover, Ahmed and Liu [11] analyzed the effects of mixed convection and mass transfer of three-dimensional oscillatory flow of a viscous incompressible fluid past an infinite vertical porous plate in presence of transverse sinusoidal suction velocity oscillating with time and a constant free stream velocity. Recently, Ahmed and Zueco [12] investigated the effects of Hall current, magnetic field, rotation of the channel and suction–injection on the oscillatory free convective MHD flow in a rotating vertical porous channel when the entire system rotates about an axis normal to the channel plates and a strong magnetic field of uniform strength is