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Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation

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Abstract The boundary layer flow and heat transfer towards a porous exponential stretching sheet in presence of a magnetic field is presented in this analysis. Velocity slip and thermal slip are considered instead of no-slip conditions at the boundary. Thermal radiation term is incorporated in the temperature equation. Similarity transformations are used to convert the partial differential equations corresponding to the momentum and energy equations into non-linear ordinary differential equations. Numerical solutions of these equations are obtained by shooting method. It is found that the horizontal velocity decreases with increasing slip parameter as well as with the increasing magnetic parameter. Temperature increases with the increasing values of magnetic parameter. Temperature is found to decrease with an increase of thermal slip parameter. Thermal radiation enhances the effective thermal diffusivity and the temperature rises.

1. Introduction

The study of laminar flow and heat transfer over a stretching sheet in a viscous fluid is of considerable interest because of its ever increasing industrial applications and important bearings on several technological processes. Examples are numerous and they include the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation processes, paper production, glass blowing, metal spinning, drawing plastic films and polymer extrusion, to name just a few [1]. Crane [2] investigated the flow caused by the stretching of a sheet. Under different physical situations, many researchers extended the work of Crane [2]. Most of the available literature deals with the study of boundary layer flow over a stretching surface where the velocity of the stretching surface is assumed linearly proportional to the distance from the fixed origin.

However, realistically stretching of plastic sheet may not necessarily be linear [3]. Flow and heat transfer characteristics past an exponentially stretching sheet has a wider applications in technology. For example, in case of annealing and thinning

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of copper wires the final product depends on the rate of heat transfer at the stretching continuous surface with exponential variations of stretching velocity and temperature distribution. During such processes, both the kinematics of stretching and the simultaneous heating or cooling have a decisive influence on the quality of the final products [4]. Magyari and Keller [5] focused on heat and mass transfer on boundary layer flow due to an exponentially continuous stretching sheet. Elbashbeshy [6] studied the flow past an exponentially stretching surface. Khan [7] and Sanjayanand and Khan [8] discussed the viscous–elastic boundary layer flow and heat transfer due to an exponentially stretching sheet. Later, Sajid and Hayat [9] find the analytic solution using homotopy analysis method and discussed the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. Recently, the effect of thermal radiation on the steady laminar two-dimensional boundary layer flow and heat transfer over an exponentially stretching sheet was reported by Bidin and Nazar [10]. Of late, El-Aziz [11] and Ishak [12] described the flow and heat transfer past an exponentially stretching sheet.

All the above mentioned studies continued their discussions by assuming the no slip boundary conditions. The no-slip boundary condition (the assumption that a liquid adheres to a solid boundary) is one of the central tenets of the Navier–Stokes theory. However, there are situations wherein this condition does not hold. Partial velocity slip may occur on the stretching boundary when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. The non-adherence of the fluid to a solid boundary, also known as velocity slip, is a phenomenon that has been observed under certain circumstances [13]. Recently, many researchers [14–18], etc. investigated the flow problems taking slip flow condition at the boundary. The fluids that exhibit boundary slip have important technological applications such as in the polishing of artificial heart valves and internal cavities. For some coated surfaces, such as Teflon, resist adhesion, the no slip condition is replaced by Navier's partial slip condition, where the slip velocity is proportional to the local shear stress. However, experiments suggest that the slip velocity also depends on the normal stress. A number of models have been advanced for describing the slip that occurs at solid boundaries. A new dimension is added to the above mentioned study by considering the effects of partial slip at the stretching wall. The study of magnetohydrodynamics boundary layer flow of a conducting fluid is also important as it finds applications in a variety of stretching sheet problems. Representative studies dealing with such effects can be found in Turkyilmazoglu [19–21].

Suction or injection (blowing) of a fluid through the bounding surface can significantly change the flow field. In general, suction tends to increase the skin friction whereas injection acts in the opposite manner. Injection or withdrawal of fluid through a porous bounding wall is of general interest in practical problems involving boundary layer control applications such as film cooling, polymer fiber coating, and coating of wires. The process of suction and blowing has also its importance in many engineering activities such as in the design of thrust bearing and radial diffusers, and thermal oil recovery. Suction is applied to chemical processes to remove reactants. Blowing is used to add reactants, cool the surface, prevent corrosion or scaling and reduce the drag.

The radiative effects have important applications in physics and engineering. The radiation heat transfer effects on different flows are very important in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effects may play an important role in controlling heat transfer in polymer processing industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, magnetohydrodynamics (MHD) accelerators, power generation systems are some important applications of radiative heat transfer from a vertical wall to conductive gray fluids.

Since no attempt has been made to analyze the effects of partial slip on MHD boundary layer flow over an exponentially stretching surface with suction or injection, so it is considered in this article. Using similarity transformations, a third order ordinary differential equation corresponding to the momentum equation and a second order differential equation corresponding to the heat equation are derived. Using shooting method numerical calculations up to desired level of accuracy were carried out for different values of dimensionless parameters of the problem under consideration for the purpose of illustrating the results graphically. The analysis of the results obtained shows that the flow field is influenced appreciably by the slip parameter in the presence of magnetic field and suction or injection at the wall. Estimation of skin friction which is very important from the industrial application point of view is also presented in this analysis. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2. Equations of motion

Consider the flow of an incompressible viscous electrically conducting fluid past a flat sheet coinciding with the plane $y = 0$ (see Fig. 1). The x-axis is directed along the continuous stretching surface and points in the direction of motion while the y-axis is perpendicular to the surface. The flow is confined to $y > 0$. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. The flow is assumed to be generated by stretching of the elastic boundary sheet from a slit with a large force such that the velocity of the boundary sheet is an exponential order of the

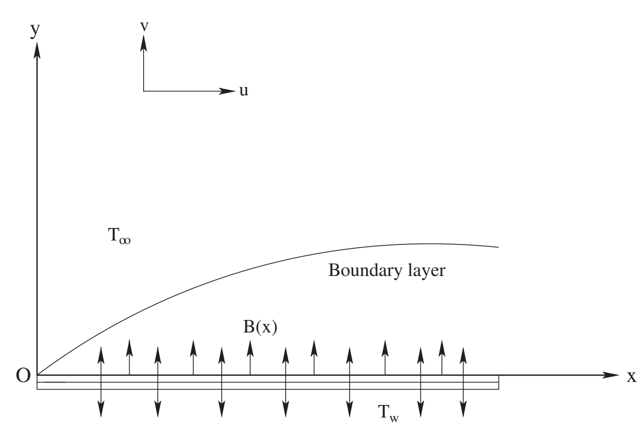

Figure 1 Sketch of physical problem.