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Effect of Coriolis force on thermomagnetic convection in a ferrofluid saturating porous medium: A weakly nonlinear stability analysis

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ABSTRACT

We investigate the influence of Coriolis force on the onset of thermomagnetic convection in ferrofluid saturating a porous layer in the presence of a uniform vertical magnetic field using both linear and weakly non-linear analyses. The modified Brinkman–Forchheimer-extended Darcy equation with Coriolis term has been used to describe the fluid flow. The linear theory based on normal mode method is considered to find the criteria for the onset of stationary thermomagnetic Convection and weakly non-linear analysis based on minimal representation of truncated Fourier series analysis containing only two terms has been used to find the Nusselt number Nu as functions of time. The range of thermal Rayleigh number R beyond which the bifurcation becomes subcritical increases with increasing Λ, Da^{-1} and Ta. The global quantity of the heat transfer rate decreases by increasing the Taylor number Ta. The results obtained, during the above analyses, have been presented graphically and the effects of various parameters on heat and mass transfer have been discussed. Finally, we have drawn the steady streamlines for various parameters.

1. Introduction

Thermomagnetic convection in magnetized ferrofluids saturating a porous layer is a topic of current technical importance since magnetic forces can be used to create circulation of coolant in small passages where natural convection is either absent or ineffective. The onset of thermomagnetic convection in a porous layer is investigated using Brinkman–Forchheimer-extended Darcy model. Most theoretical studies of ferroconvection in non-porous and porous domains have concentrated on the linearized equations and the studies undertaken on nonlinear convection are very sparse. Since the size of the ferromagnetic particles is small, ferrofluids can flow through porous media such as natural sediments or fractured rock due to gravitational, pressure gradient, capillary, and magnetic forces (Borglin et al. [1] and Oldenburg et al. [2]).

Schwab [3] has reported an analytical weakly nonlinear analysis of convective heat transfer in a ferrofluid of infinite magnetic susceptibility between free horizontal surfaces. Blennerhassett et al. [4] have analyzed weakly nonlinear thermo-convective stability of a ferrofluid, confined between rigid horizontal plates at different temperatures and subjected to a strong uniform external magnetostatic field in the vertical direction. Nonlinear convective roll cells that develop in thin layers of magnetized ferrofluids heated from above are examined in the limit as the wave number of the cells becomes large by Russell et al. [5]. Kaloni and Lou [6], Sunil and Mahajan [7] have performed nonlinear stability analysis of magnetized ferrofluid bounded by stress free boundaries and heated from below using the energy method. Recently, Laroze et al. [8] have reported theoretical and numerical results on thermally driven convection of a magnetic suspension by performing a truncated Galerkin expansion finding that the system can be described by a generalized Lorenz model.

The use of energy method to construct stability thresholds is crucial to assess whether the linear theory accurately encapsulates the physics of onset and the behavior of nonlinear ferroconvection in a porous medium. Straughan [9] has shown that the global nonlinear stability threshold for convection with a thermal nonequilibrium model is exactly the same as the linear instability boundary. Thermoconvective instability of magnetized ferrofluid saturating a porous medium has been studied using the global nonlinear stability analysis by Qin and Chadam [10] Sunil and Mahajan [11], while similar analysis has been carried out on the problem to include the local thermal nonequilibrium (LTNE) effects by Sunil et al. [12].

Blums et al. [13] have dealt with transport properties of ferrofluid nanoparticles in non-isothermal capillary-porous layer.

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Experimentally they have established that the temperature difference, which is applied across the layer, induces a thermo-osmotic pressure gradient directed toward increasing temperature. The measurement results are interpreted in a frame of phenomenology of linear irreversible thermodynamics.

Jyoti Prakash [14] has proved that the principle of exchange of stabilities is not, in general valid, for the case of free boundaries, in ferromagnetic convection, in a porous medium in the presence of a uniform vertical magnetic field and uniform rotation about the vertical axis and a sufficient condition is derived for the validity of this principle. Sekar et al. [15] have analyzed the Soret-driven ferro-thermo-convective instability of multicomponent fluid in an anisotropic porous medium heated from below and salted from above using the Brinkman model for various values of anisotropic parameter. Both stationary as well as oscillatory modes are considered in the study. The vertical anisotropy is shown to destabilize the system.

Jyoti Prakash and Sanjay Gupta [16] have investigated on arresting the complex growth rates in ferromagnetic convection with magnetic field dependent viscosity in a rotating ferrofluid layer. Straughan et al. [17] have analyzed Rotating porous convection with prescribed heat flux. Sekar et al. [18] have Studied the ferroconvection in fluids saturating a rotating densely packed porous medium. Nanjundappa et al. [19] have analysed penetrative ferroconvection via Internal Heating in a Saturated Porous Layer with Constant Heat Flux at the Lower Boundary. Nanjundappa et al. [20] was investigated the onset of coupled Bénard–Marangoni convection in a horizontal layer of ferrofluid with viscosity depending exponentially on temperature is investigated. The lower rigid and the upper free boundaries are assumed to be insulated to temperature perturbations and the free boundary at which the surface tension effects are accounted for is assumed to be non-deformable. The resulting eigenvalue problem is solved numerically using the Galerkin technique and also analytically by a regular perturbation technique with a wave number as a perturbation parameter. The analytical and numerically computed results are found to be in concurrence. The combined effect of magnetic number M_1 and the viscosity parameter B is to reinforce together and to hasten the onset of Bénard–Marangoni ferroconvection compared to their presence in isolation. Nonetheless, the effect of increasing B also shows initially some stabilizing effect on the system depending on the strength of magnetic and buoyancy forces. In addition, the nonlinearity of fluid magnetization is found to have no influence on the criterion for the onset of Bénard–Marangoni ferroconvection.

Nonetheless, a different approach is followed in the present paper to study nonlinear thermomagnetic convection in a ferrofluid saturated porous medium. Instead of grappling with the full problem a simplified extended Lorenz model, put forward by Russell et al. [5], which reproduces qualitative features of the full system with remarkable fidelity is considered. This model problem, consisting of seven coupled nonlinear autonomous ordinary differential equations, are solved with sufficient accuracy by a combination of analytical and numerical techniques. The stability of bifurcating equilibrium solution is analyzed and heat transport is calculated in terms of Nusselt number which is missing in the previous studies.

2. Formulation of the problem

We consider an initially quiescent incompressible constant viscosity ferromagnetic fluid saturated horizontal porous layer of characteristic thickness d in the presence of a uniform applied magnetic field H_0 in the vertical direction. The system is considered to be rotating about an axis with non-uniform rotation speed. In particular, we assume that the rotation speed is varying sinusoidally with time. The lower surface is held at constant temperature T_l, while the upper surface is at $T_u\ (<T_l)$. A Cartesian co-ordinate system (x, y, z) is used with the origin at the bottom of the porous layer and the z-axis directed vertically upward in the presence of gravitational field. The flow in the rotating ferrofluid saturating a porous medium is described by modified Brinkman–Forchheimer-extended Darcy equation with fluid viscosity different from effective viscosity and the Boussinesq approximation on the density is made.

The basic equations governing the flow of an incompressible ferrofluid saturated porous medium under Oberbeck–Boussinesq approximation are:

$$\nabla\times\vec{q}=0. \tag{1}$$

$$\begin{aligned}\frac{\rho_0}{\varepsilon}\left[\frac{\partial\vec{q}}{\partial t}+\frac{1}{\varepsilon}(\vec{q}\times\nabla)\,\vec{q}\right] &= -\nabla\left(p_f+\frac{\rho_0}{2}\left|\vec{\Omega}\times\vec{r}\right|^2\right)+\rho\,\vec{g}-\frac{\mu_f}{k}\vec{q}+\tilde{\mu}_f\,\nabla^2\vec{q}.\\ &\quad+\mu_0(\vec{M}\times\nabla)\vec{H}+2\rho_0(\vec{q}\times\vec{\Omega}).\end{aligned} \tag{2}$$

$$\varepsilon\left[\rho_0 C_{V,H}-\mu_0\vec{H}\times\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\right]\frac{DT}{Dt}+(1-\varepsilon)\,(\rho_0 C)s\,\frac{\partial T}{\partial t}+\mu_0 T\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\times\frac{D\vec{H}}{Dt}=k_t\nabla^2 T. \tag{3}$$

$$\rho=\rho_0\ [1-\alpha(T-T_0)] \tag{4}$$

$$\nabla\times\vec{B}=0 \tag{5a}$$

$$\nabla\times\vec{H}=0 \text{ or } \vec{H}=\nabla\phi \tag{5b}$$

here, $P=p+\rho_0|\vec{\Omega}\times\vec{r}|^2/2$ is the total pressure, $\vec{q}$ is the velocity of fluid particle, $\vec{\Omega}$ is the angular velocity, $\vec{r}$ is the position vector, p is the fluid pressure, ρ is the fluid density, ρ_0 is the reference density, $\vec{M}$ is the magnetization, $\vec{H}$ is the magnetic field intensity, $\vec{B}$ is the magnetic flux density, μ_0 is the magnetic permeability of vacuum, k_t is the thermal conductivity, $C_{V,H}$ is the specific heat at constant volume and magnetic field, α is the thermal expansion coefficient and $T_a=(T_l+T_u)/2$ is the average temperature, μ_f and $\tilde{\mu}_f$ are the dynamic and effective viscosities, k is the permeability of the porous medium and ε is the porosity of the porous medium, $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial z^2$ is the Laplacian operator.

Further, $\vec{B}$, $\vec{M}$ and $\vec{H}$ are related by

$$\vec{B}=\mu_0\,(\vec{M}+\vec{H}) \tag{6}$$

The local magnetization $\vec{M}$ is considered to be a function of $\vec{H}$ and T in the form (Russell et al. [5])

$$\vec{M}=\chi(T)\vec{H} \tag{7}$$

where $\chi(T)$ is the magnetic susceptibility. This expression applies quite generally when the magnetic field is weak and we assume that $\chi(T)$ can be written in a form similar to the density as

$$\chi(T)=\chi_0+\left(\frac{d\chi}{dT}\right)_{T_a}(T-T_a). \tag{8}$$

The basic state is assumed to be quiescent and is given by

$$\vec{q}_b=0 \tag{9}$$

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