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On computation of relaxation constant α in Landau–Lifshitz–Gilbert equation

S.O. Gladkov, S.B. Bogdanova

Moscow Aviation Institution (National Research University) (MAI) 4 Volokolamskoe shosse, A-80, GSP-3, Moscow 125993, Russia

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A B S T R A C T

Due to the quasi-classical kinetic equation (QKE) for the magnon distribution function to calculate the velocity of the domain wall motion V in magnetic fields $H > H_a$, where H_a- magnetic anisotropy field. Based on the comparison of this formula for $V$the analytic expression of relaxation constant α in Landau–Lifshitz–Gilbert equation was found. We used the detected correlation between the system's entropy and the environment's resistance force, and obtained an expression for the spin-lattice braking force that is applied to the moving domain wall. We calculated the mobility ratio of the domain wall.

1. Introduction

It is notable that despite the century-long history of studies concerning the properties of the domain structure (for example, see papers [1,2]) and monographs [3–7], for some reason there are no papers on quasi-classical calculation of relation between velocity of the domain wall $\vec{V}$ and magnetic field $\vec{H}$. Though many researchers calculated the speed of the domain wall, the classical work in this field is apparently the work by Thiele [1], where it was obtained the general formula for the speed of domain wall, which is valid for any speed value, not just for low ones. Since the ultimate goal of our work is to calculate phenomenologically introduced constant α which is involved in Landau–Lifshitz–Gilbert $\partial\vec{M}/\partial t = \gamma_e[\vec{H}_{ef} \times \vec{M}] + \alpha/M_0[\vec{M} \times (\partial\vec{M}/\partial t)]$, where $\vec{M}-$ magnetization density (magnetic moment per unit volume), $\gamma_e = ge/2mc$, $g-$ Lande factor, $e-$ electron charge, $m-$ its mass, $c-$ light velocity, M_0- spontaneous magnetization of ferromagnetic, $\vec{H}_{ef}-$ effective magnetic field, influencing magnetic moment density $\vec{M}$ and being defined as functional derivative from free energy of magnetic F according to magnetization $\vec{M}$, which negative, i.e. $\vec{H}_{ef} = -\delta F/\delta\vec{M}$, then it would be quite logical to preliminary study correctness of our approach.

Generally speaking, only the correspondence between theoretical calculations and experimental data can serve as the main criterion of the correctness of the results below. In fact, the estimate value of motion velocity of the domain wall in the external magnetic field, expressed as motion μ, provided that μ is calculated analytically, allows connecting $\vec{V}$ to $\vec{H}$ and so finding the sought proportionality factor. Since, according to known dependence $V = (ge\delta/2mc\alpha)H$, where $\delta = a\sqrt{J_{ex}/\beta\mu_e M_0}-$ thickness of the domain wall, $a-$ atom spacing, $\mu_e = \hbar\gamma_e-$ Bohr magneton, then after setting $\mu_{theor.}$ equal to proportionality factor $V/H = ge\delta/2mc\alpha$, we can immediately find the sought analytic expression for constant α, involved in LLG equation. Thus, the problem becomes quite clear, and we can begin solving it, that is, computing velocity V.

Preliminary we should say a few words concerning the theory of domains. Weiss introduced the concept of the domain structure in 1907, and L.D. Landau and E.M. Lifshitz proposed the full domain thermodynamic theory in 1935. Under this model, the average domain size b and thickness of domain wall $\delta = a\sqrt{J_{ex}/\beta\mu_e M_0}$ were calculated, as a result of competition between the exchange interaction of magnetic atoms J_{ex} with anisotropy energy. $\beta\mu_e M_0$. At that, the average size of domain b was found under minimum condition of free energy by this parameter. In the result it appeared that $b = \sqrt{l\delta}$, where $l-$ size of ferromagnetic along y axis (see Fig. 1) which is perpendicular to the figure.

As specified in [6], the motion velocity of the domain wall can be defined by simple formula $V = \mu H$, where $\mu-$ mobility of the wall in the magnetic field H. Here, the value of the mobility coefficient is as follows: $\mu = e\tau^*/m$, where $e-$ electron charge, $m-$ its mass, and τ^*- certain invariable having time dimensionality. Its relation to constant α can be easily found if applying classic formula for domain wall velocity $V = (ge\delta/2mc\alpha)H$. From this comparison we can see that $\tau^* = g\delta/2c\alpha$. Based on the specified formula, for values $e = 4.48 \cdot 10^{-10} SGS$, $m = 0.9 \cdot 10^{-27}$g, $c = 3 \cdot 10^{10}$ cm/s, $g = 6$, $\delta = 10^{-6}$cm, $\alpha = 10^{-2}$, $\tau^* = 10^{-14}$ s,and the velocity value is appeared to be approximately 10^3 H. If the magnetic field is about one Ersted, the velocity of the domain wall is about 10 m/s.

E-mail addresses: sglad@newmail.ru (S.O. Gladkov), sonjaf@list.ru (S.B. Bogdanova).

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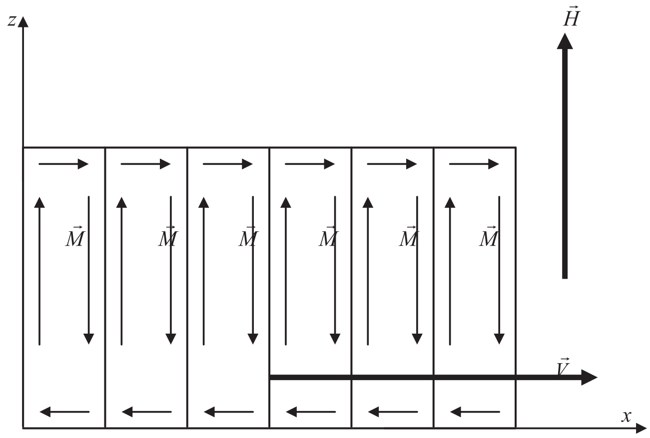

Fig. 1. Schematic images of the planar domain wall in form of alternate plane-parallel zones. The speed of each domain is directed along axis *x*. External magnetic field is oriented along axis *z*.

Theoretically such velocity value is quite acceptable if you remember the Brockhaus experience, when the motion of domain walls during their destruction in a magnetic field exceeding an anisotropy field, is accompanied by the rustle occurring as a result of chaotic motion, which lasts for quite visible time from a range of human hearing, right up to seconds.

At the same time if you pay attention to the numerical value of αcoefficient, which we would accept for evaluation, it is possible to make conclusion that the order of its value is found out to be a little undervalued and the most acceptable value of Gilbert constant should be seemingly around $\alpha = 1 \div 10$.

These arguments lead us back to this problem again and consider calculating constant α and velocity Vas a certain function of the physical parameters of the ferromagnetic dielectric.

We would like to note, that the main objective of the work is to calculate the relaxation constant α, and not the speed of domain wall $\vec{V}$. On the other hand, our problem requires to determine the link between $\vec{V}$ and the magnetic field $\vec{H}$. It means that we need to calculate the mobility ratio μ, which, in its turn, is related to the Hilbert constant α. As a result we will solve the problem. Its solution is based on the use of quasiclassical kinetic equation that will let us calculate the system's entropy and correlate its with the environment's resistance force. This problem was first solved in the work [8] therefore results of the article [8] will help us determine the desired relation $\mu(\alpha)$.

2. Formulation on the problem and Its solution

Imagine easy-axis ferromagnetic dielectric, the easy magnetization axis of which is chosen as axis z. We will direct external magnetic field H also along this axis, and present the model domain structure in form of planar-parallel alternate zones demonstrated in Fig. 1. The width and height of domain is denoted by letter b, and the thickness of the domain wall is denoted by δ (see the text above and [2]).

The order of magnitude of magnetic anisotropy constant β is easy to evaluate based on the following simple considerations. In fact, if the spin (orbital interaction of magnetic atoms) is (see [3]) $V_{SL} = B\vec{S}\vec{L}$, where $B-$ spin energy of orbital interaction, $\vec{S}$ – spin operator, and $\vec{L}$ – orbital moment operator of atom, then the spin energy of orbital connection must be equal around $B \approx \mu_e^2/a^3 = 1/4(\bar{u}/c)^2\hbar/ma^2$, where the average speed of electrons in an atom is $\bar{u} = e/\sqrt{ma}$. If we use $\bar{\varepsilon}_{orb}$ to denote the average energy of the orbital motion of electron, which can be defined as $\bar{\varepsilon}_{orb} = \hbar/\tau$, where $\tau-$ the orbit time of electron around the core, while $\tau = 2\pi a/\bar{u}$, and $\bar{u} = e/\sqrt{ma}$, then we get that $\bar{\varepsilon}_{orb} = \hbar e/2\pi a\sqrt{ma}$ and according to the general principals of perturbation theory [4], the anisotropy energy can be presented as $V_a = \langle B\vec{S}\vec{L}\rangle^2/2\bar{\varepsilon}_{orb}$, where the angular brackets mean averaging over fast orbital motion. As a result we find out that $V_a = \langle B\vec{S}\vec{L}\rangle^2/2\bar{\varepsilon}_{orb} = B^2 S_z^2/2(2L+1)\bar{\varepsilon}_{orb}\sum_{M_L=-L}^{L} M_L^2 = B^2 L(L+1)/6\bar{\varepsilon}_{orb} S_z^2$. Next, to consider the opportunity of temperature phase transition of ferromagnetic from the easy-axis state to the easy-plane one, we present average spontaneous magnetization of an atom $M_0 = (\mu_e/a^3)S$ as $M_0 = (\mu_e/a^3)S(\sqrt{(T/T_{cr})-1})$, where $T_{cr}-$ temperature of phase transition "easy-axis" – "easy-axis" (see [3, p. 201]). In view of these circumstances the sought anisotropy energy will be

$$V_a = \frac{\beta_0}{2}\left(\frac{T}{T_{cr}} - 1\right)S_z^2, \tag{1}$$

where the sought constant will be

$$\beta_0 = \frac{\pi\mu_e^4 L(L+1)\sqrt{ma}}{3a^5 e\hbar}. \tag{2}$$

If we insert the values of all parameters here, viz, $\mu_e = 1.66\cdot 10^{-20}SGS, a = 3\cdot 10^{-8}\text{cm}, e = 4.48\cdot 10^{-10}SGS$, $m = 0.9\cdot 10^{-27}\text{g}, L = 2$, then we get the following estimate

$$\beta_0 = 2.27\cdot 10^{-16}SGS. \tag{3}$$

It means approximately one degree Kelvin.

As a rule, when solving the relevant problems of the magnetism theory [5,6], in terms of the macroscopic examination, it is not anisotropy constant which is used but its value per unit volume.

If we denote this constant by K as in [3], which can be defined as $K = \beta_0/a^3$, then instead of (2) we will get

$$K = \frac{\pi\mu_e^4 L(L+1)\sqrt{ma}}{3a^8 e\hbar}. \tag{4}$$

According to the estimate (3) for $a = 3\cdot 10^{-8}$cm we will get

$$K \approx 8.4\cdot 10^6 SGS. \tag{5}$$

Therefore, the extrapolated to zero constant of anisotropy K, as it follows from the above considerations, is very well coherent with the results of its measurements, in particular for hexagonal cobalt, which is also noted in [3, p. 201].

All the above considered, the total free energy of the easy-axis ferromagnetic (i.e. as $T < T_{cr}$) can be written down as follows:

$$F_{ЛО} = \int_\Omega \left[\frac{\alpha}{2}\left(\frac{\partial M_i}{\partial x_k}\right)^2 - \frac{K}{2M_0^2}\left(1 - \frac{T}{T_{cr}}\right)M_z^2 + \frac{K_1}{4M_0^4}M_z^4 - M_z H\right]dV, \tag{6}$$

where K_1 – magnetic anisotropy constant based on the following magnetization anharmonicities M_z.

In case the temperature exceeds the critical point, i.e. $T > T_{cr}$, as can be seen from (6), it is energetically beneficial for an easy-axis ferromagnetic to transit to anisotropy state like "easy-plane" with free energy.

$$F_{ep} = \int_\Omega \left[\frac{\alpha}{2}\left(\frac{\partial M_i}{\partial x_k}\right)^2 - \frac{K}{2M_0^2}\left(\frac{T}{T_{cr}} - 1\right)(M_x^2 + M_y^2) + \frac{K_1}{4M_0^4}M_z^4 - M_z H\right]dV. \tag{7}$$

where magnetization now lies in plane $x-y$.

As it can be seen from (6) and (7), jump in entropy at point $T = T_{cr}$ equals to $\Delta S = \frac{K}{2}\Omega$, and it is the point of first-order phase transition.

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