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Influence of heat transfer on MHD flow in a pipe with expanding or contracting permeable wall

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Abstract The present study investigates the effects of heat transfer on MHD laminar viscous flow in a pipe with expanding or contracting permeable wall. The pipe wall expands or contracts uniformly at a time dependent rate. The governing equations are reduced to ordinary differential equations by using a similarity transformation. An analytical approach, namely the homotopy analysis method (HAM) is applied in order to obtain the solutions of the ordinary differential equations. The effects of various emerging parameters on flow variables have been discussed numerically and explained graphically. Further, we find a good agreement between the HAM solutions and solutions already reported in the literature.

1. Introduction

A lot of attention has been given to the studies pertaining to laminar flow in a porous pipe or channel with expanding or contracting walls due to their wide applications in technological as well as biological flows, for example in the transport of biological fluids through expanding or contracting vessels, the synchronous pulsation of porous diaphragms, the air circulation in the respiratory system and the regression of the burning surface in solid rocket motors [1–7]. The viscous flow inside an impermeable tube of contracting cross section was first examined by Uchida and Aoki [8]. Therein, the Navier–Stokes equations for a semi-infinite tube were reduced to a single differential equation. They investigated a similar solution for the unsteady flows produced by a singe contraction or expansion of the wall of a semi-infinite circular pipe. Goto and Uchida [9] carried out a theoretical analysis of the unsteady flow in a semi-infinite expanding or contracting circular pipe into which an incompressible fluid is injected or sucked in through the wall surface. Bujurke et al. [10] have studied the unsteady flow in a contracting or expanding pipe by using a computer extended series solution. Boutros et al. [11] have applied the Lie-group method for unsteady flows in a semi-infinite expanding or contracting pipe with injection or suction through a porous wall. In their investigation the Lie-group method was applied to the equations of the motion for determining symmetry reductions in partial differential equations, the resulting fourth order differential equation was then solved using

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small-parameter perturbations and the results were compared with numerical solutions using a shooting method coupled with a Runge–Kutta scheme. Xinhui et al. [12] have analyzed the problem of laminar flow in a porous pipe with suction at slowly expanding or contracting wall. Majdalani and Zhou [13] studied moderate to large injection and suction driven channel flows with expanding or contracting walls. In this investigation the governing equation is first integrated and the resulting third-order differential equation is solved using the method of variation in parameters, for the large injection case. For the large suction case, the governing equation is first simplified near the wall and then solved using successive approximations.

There has been growing interest in studying the magnetohydrodynamic (MHD) flow and heat transfer characteristics of electrically conducting fluids because of many practical applications such as in MHD flowmetry, MHD power generation MHD pumps, high temperature plasmas, chemical processing equipment, power generation systems and cooling of nuclear reactors [14–18]. Hayat et al. [19] have obtained explicit analytical solutions for MHD pipe flow of a fourth grade fluid. Xinhui et al. [20] discussed the unsteady flow in a porous channel with expanding or contracting walls in the presence of a transverse magnetic field using the singular perturbation method. Turkyilmazoglu [21] obtained exact solutions for the steady Navier–Stokes equations governing the incompressible viscous Newtonian electrically conducting fluid flow due to rotating disk. Makinde et al. [22] have examined wall driven steady flow of a viscous fluid and heat transfer in a uniformly porous tube using perturbation series. Nakhi and Chamkha [23] analyzed the conjugate natural convection around a finned pipe in a square enclosure with internal heat generation. Recently, Srinivas et al. ([24] several references therein) studied the thermal diffusion and diffusion thermo effects in a two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability. Reddy et al. [25] analyzed the influence of heat transfer and chemical reaction on asymmetric laminar flow between two slowly expanding or contracting walls using a double perturbation in the permeation Reynolds number and the wall expansion ratio. More recently, Srinivas et al. [26] have examined the effects of mass transfer and chemical reaction on laminar flow in a porous pipe with expanding or contracting wall by using the homotopy analysis method.

A literature survey reveals that no attempt regarding heat transfer effects on MHD flow of viscous fluid in a porous pipe with expanding or contracting wall has been made so far. Such a study is of great value in biological and engineering research. Hence the main aim of this work is to study the effect of heat transfer on MHD viscous flow in a porous pipe with expanding or contracting wall. The governing equations in cylindrical coordinates are introduced and transformed into ordinary differential equations using similarity transformations and then solved using a powerful technique recently developed by Liao [27] namely the homotopy analysis method (HAM). This technique has been applied successfully to many interesting problems ([28-35]). The features of the flow characteristics are analyzed by plotting graphs and are discussed in detail. The present paper is organized in the following manner. In Section 2, details of the mathematical formulation are presented. Sections 2.1 and 2.2 include the solution procedure of the problem. Numerical results and discussion are given in Section 3 and the conclusions have been summarized in Section 4.

2. Formulation of the problem

Consider the laminar and incompressible electrically conducting fluid flow in a porous pipe of a semi-infinite length with an expanding or contracting wall. The radius of the pipe is $a(t)$. The wall has equal permeability and expands or contracts uniformly at a time dependent rate $\dot{a}(t)$. A magnetic field of uniform strength B_0 is applied perpendicular to the wall. A coordinate system can be chosen with the origin at the center of the pipe as shown in Fig. 1. Take the $\hat{z}$ coordinate axis parallel to the pipe wall and the $\hat{r}$ coordinate axis perpendicular to the wall. Under these assumptions the governing equations are ([8], [11], and [12])

$$\frac{\partial \hat{u}}{\partial \hat{z}} + \frac{\partial \hat{v}}{\partial \hat{r}} + \frac{\hat{v}}{\hat{r}} = 0, \tag{1}$$

$$\frac{\partial \hat{u}}{\partial t} + \hat{u}\frac{\partial \hat{u}}{\partial \hat{z}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{r}} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{z}} + \nu\left(\frac{\partial^2 \hat{u}}{\partial \hat{z}^2} + \frac{\partial^2 \hat{u}}{\partial \hat{r}^2} + \frac{1}{\hat{r}}\frac{\partial \hat{u}}{\partial \hat{r}}\right) - \frac{\sigma B_0^2}{\rho}\hat{u}, \tag{2}$$

$$\frac{\partial \hat{v}}{\partial t} + \hat{u}\frac{\partial \hat{v}}{\partial \hat{z}} + \hat{v}\frac{\partial \hat{v}}{\partial \hat{r}} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{r}} + \nu\left(\frac{\partial^2 \hat{v}}{\partial \hat{z}^2} + \frac{\partial^2 \hat{v}}{\partial \hat{r}^2} + \frac{1}{\hat{r}}\frac{\partial \hat{v}}{\partial \hat{r}} - \frac{\hat{v}}{\hat{r}^2}\right), \tag{3}$$

$$\frac{\partial T}{\partial t} + \hat{u}\frac{\partial T}{\partial \hat{z}} + \hat{v}\frac{\partial T}{\partial \hat{r}} = \frac{\kappa}{\rho c_p}\left(\frac{\partial^2 T}{\partial \hat{z}^2} + \frac{\partial^2 T}{\partial \hat{r}^2} + \frac{1}{\hat{r}}\frac{\partial T}{\partial \hat{r}}\right) \tag{4}$$

where $\hat{u}, \hat{v}$ are the components of velocity along the $\hat{z}$ and $\hat{r}$ directions respectively, ρ is density, $\hat{p}$ is dimensional pressure, t is time, ν is kinematic viscosity, σ is electrical conductivity, B_0 is the strength of applied magnetic field, c_p is specific heat at constant pressure, κ is thermal conductivity, and T is the temperature of the fluid.

The boundary conditions are as follows:

$$\hat{u} = 0, \quad \hat{v} = -v_w = -A\dot{a}, \quad T = T_w \quad \text{at } \hat{r} = a(t) \tag{5}$$

$$\frac{\partial \hat{u}}{\partial \hat{r}} = 0, \hat{v} = 0, \quad \frac{\partial T}{\partial \hat{r}} = 0 \quad \text{at } \hat{r} = 0 \tag{6}$$

$$\hat{u} = 0, \hat{v} = 0 \quad \text{at } \hat{z} = 0 \tag{7}$$

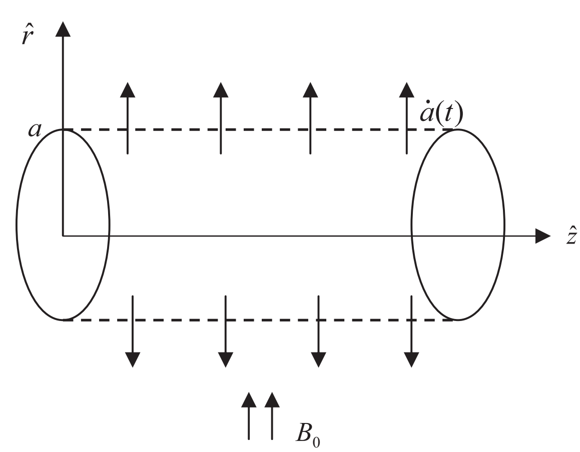

Figure 1 Porous pipe with expanding or contracting wall.