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Stress concentration analysis in functionally graded plates with elliptic holes under biaxial loadings

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Abstract Stress concentration factors (SCFs) at the root of an elliptic hole in unidirectional functionally graded material (UDFGM) plates under uniaxial and biaxial loads are predicted. ANSYS Parametric Design Language (APDL) was used to build the finite element models for the plates and to run the analysis. A parametric study is performed for several geometric and material parameters such as the elliptic hole major axis to plate width ratio, the elliptical shape factor, the gradation direction of UDFGM. It is shown that, SCF in the finite plate can be significantly reduced by choosing the proper distribution of the functionally graded materials. The present study may provide designers an efficient way to estimate the hole effect on plate structures made of functionally graded materials.

1. Introduction

Functionally graded material (FGM) is a new kind of inhomogeneous composite. It possesses continuously varying microstructure and mechanical properties. The main advantage of FGM is that no internal boundaries exist and the interfacial stress concentrations can be avoided. Furthermore, functionally graded materials (FGMs) can be designed to achieve particular desired properties and the gradation in properties of the material can optimize stress distribution. Nowadays, there have been increasingly many modern engineering applications of FGMs, such as space shuttle, rocking-motor casings, and packaging materials in microelectronic industry [1].

Stress concentrations around cutouts have great practical importance because they are normally the cause of failure. Stress concentration factor (SCF), K_t, defined as the ratio of the maximum stress in the presence of a geometric irregularity or discontinuity to the stress away from the effect of such irregularity, i.e. applied stress. Majority of the studies performed for the SCF have treated isotropic, orthotropic or composite plates. However, due to the abrupt changes in the material properties of the laminated composite structures in the transverse direction and subsequently, possibility of local failure occurrence, functionally graded materials were used as alternative materials in some applications.

Wu and Mu [2] proposed a simple computation method based on the scale factors (SFs) to estimate the stress concentration factors (SCFs) of finite-width isotropic/orthotropic plates/cylinders with a circular cutout and under uniaxial or biaxial tension. Ukadgaonker and Kakhandki [3] analyzed

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the stresses in an orthotropic plate with an irregular shaped hole under different in-plane loading conditions and had well matching with the FEM solutions. Rao et al. [4] gave analytical solution to get the stress distribution around square and rectangular cutouts in symmetric laminates as well as in isotropic plates. Darwish et al. [5,6] investigated the in-plane SCF in countersunk rivet holes in orthotropic laminated plates under uniaxial tension load using finite element analysis. They formulated a general parametric equation for the maximum SCF. The general stress functions for determining the stress concentration around circular, elliptical and triangular cutouts in laminated composite infinite plate subjected to arbitrary biaxial loading at infinity using Muskhelishvili's complex variable method obtained by Sharma [7].

Toubal et al. [8] compared their experimental results of the tensile strain field around circular hole in a composites plate with the predictions of a theoretical model previously developed by Lekhnitskii's and a finite element study. For a plate containing a hole subjected to uniaxial tension or out-of-plane bending, Yang et al. [9,10] and Yang [11] examined the sensitivity of the stress and strain concentration factors to plate thickness as well as the Poisson's ratio or moment ratio. Moreover, the stress concentration and the influence of Poisson's ratio on the thickness-dependent SCF along the root of elliptic holes in elastic plates subjected to tension investigated by She and Guo [12] and Yu et al. [13] using 3D FEM and some empirical formulae obtained by fitting the numerical results. Sitzer and Stavsky [14] found that the SCFs were to be quite sensitive to changes in plate anisotropy and heterogeneity, direction of external tensile force and form of hole in symmetrically laminated anisotropic plates under tension.

Arjyal et al. [15] employed the remote laser Raman spectroscopy to measure the stress concentration arising in a composite Kevlar 49 fiber/epoxy composite plate containing a circular hole under different strain levels until fracture. They found that both analytical and numerical models predicted maximum values of stress concentration that were very close to that determined experimentally. Additionally, extensive experimental and numerical studies were done by Rhee et al. [16,17] for optimizing the size and location of auxiliary holes to minimize stress concentrations in uniaxially-loaded orthotropic materials.

Kubair and Bhanu-Chandar [18] performed a parametric study on functionally graded panels with circular hole under uniaxial tension by varying the functional form and the direction of the material property gradation and showed that the SCF was reduced when Young's modulus progressively increased away from the hole. Mohammadi et al. [19] analyzed the effect of nonhomogeneous stiffness and varying Poisson's ratio upon the SCFs at circular hole in an infinite plate made of a functionally graded material subjected to uniform biaxial tension and pure shear.

From the above discussion, it can be noted that, limited researches have been performed to analyze the stress concentration due to circular hole in a functionally graded plate. Moreover; to our knowledge; there are no studies carried out to obtain the stress concentration due to elliptic hole in a functionally graded plate. Therefore, the main objective of the present research is to perform stress analysis of a finite functionally graded material (FGM) plate with an elliptic hole under uniaxial and biaxial loading conditions. The two-dimensional distributions of stresses near the elliptic hole are analyzed. The sensitivity of the SCFs to FGM properties, such as gradation direction and composition parameter, and the geometric parameters is examined.

2. Functionally graded material (FGM)

Functionally graded material (FGM) characterized by smoothly variation in properties with spatial position to efficiently responds to the surrounding mechanical loads [20,21]. It comprises a multi-phase material with volume fractions of the constituents varying gradually in a pre-determined and designed profile. The concept of a FGM tries to overcome the drawbacks of composites by resolving the problem of the material property mismatch especially at the interfaces [22–24]. The advances in material synthesis technologies have encouraged the development of FGM with promising applications in aerospace, transportation, energy, electronics and biomedical engineering [25]. Significant advances in fabrication and processing techniques have made it possible to produce FGMs with complex properties and shapes using computer-aided manufacturing techniques [26]. FGM manufacturing processes include exposure to ultraviolet radiation, controlled suspension of particles in polymer matrices, high-speed centrifugal casting, depositing layers on a substrate, etc. [27,28].

2.1. Volume fractions and rule of mixtures of one-dimensional FGM

Consider a plate of FGM with porosity that functionally graded from two materials. The subscripts 1 and 2 denote *material 1* and *material 2* of the basic constituents, respectively. V_1 and V_2 are the volume fractions of *material 1* and *material 2* respectively. The volume fractions are distributed over the x-direction (horizontal distribution, Fig. 1a), according to the following relations [29,30]:

$$V_1 = \left(\frac{x}{W}\right)^m \tag{1}$$

$$V_2 = (1 - V_1) \tag{2}$$

where W and x are the plate width and the horizontal position of different points along it respectively. Moreover, m is a parameter that controls the composition variation through the plate. For *material 1* rich composition $m < 1$, while for *material 2* rich composition $m > 1$. The variation of *material 1* and *material 2* composition is linear if $m = 1$. The porosity p of the FGM may be represented for horizontal distribution model by [29]:

$$p = A\left(\frac{x}{W}\right)^n \left[1 - \left(\frac{x}{W}\right)^z\right] \tag{3}$$

$$\text{where } 0 \le A \le \frac{((n+z)/n)^n}{1 - (n/(n+z))^z} \tag{4}$$

A, n and z are arbitrary parameters that control the porosity and equal to 0.1, 1 and 1 respectively [30].

FGM effective properties, with porosity and continuously graded profile, are determined by employing the suspended spherical grain model. It was derived on the base of the assumption that the granular phase is in a matrix phase. Subsequent relations give the rules of mixture for the elastic modulus [29].