ARTICLE IN PRESS

Journal of Magnetism and Magnetic Materials ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Diffusion in active magnetic colloids

R. Taukulis, A. Cebers*

University of Latvia, Zeļļu-8, Rīga, LV-1002, Latvia

ARTICLE INFO

Keywords:
Active system
Magnetotactic bacteria
Circle swimmer
Diffusion

ABSTRACT

Properties of active colloids of circle swimmers are reviewed. As a particular example of active magnetic colloids the magnetotactic bacteria under the action of a rotating magnetic field is considered. The relation for a diffusion coefficient due to the random switching of the direction of rotation of their rotary motors is derived on the basis of the master equation. The obtained relation is confirmed by the direct numerical simulation of random trajectory of a magnetotactic bacterium under the action of the Poisson type internal noise due to the random switching of rotary motors. The results obtained are in qualitative and quantitative agreement with the available experimental results and allow one to determine the characteristic time between the switching events of a rotary motor of the bacterium.

0. Introduction

Properties of random trajectories of the so-called circle swimmers have been studied by various groups of researchers [1–5]. Various kinds of these swimmers and their properties have been investigated. Effective diffusion coefficients of circle swimmers subjected to thermal noise are calculated in [1,2]. Circle swimmers with an asymmetric L-shape self-propelling due to the self-phoresis at illumination by light were synthesized and studied in [4]. Circular dynamics of slightly bent self-propelling rods switching their direction of circulation due to thermal fluctuations are studied in [5].

A specific example of circle swimmers is magnetotactic bacteria, which in a rotating magnetic field in the absence of noise moves along circular trajectories, if the frequency of the rotating field is less than a critical value [6]. A random walk of the centers of circular trajectory is observed due to random switching of the direction of rotation of their flagella [6,7]. Here the properties of this random walk are studied on the basis of the master equation [5,8] for the probability density distribution function $f(\vec{x},\vec{n},t)$ of position of the bacterium $\vec{x}$ and direction of its magnetic moment $\vec{n}$. Bacteria moves in $\vec{n}$ direction or opposite to it. It is illustrated that this method may be applied also to circle swimmers of other types.

1. Master equation

Let us consider magnetotactic bacteria which moves in the direction of their magnetic moment $\vec{n}$ or opposite to it due to random switching of the direction of rotation of a rotary motor. Since in the synchronous regime the magnetic moment rotates synchronously with the field, so does the velocity of the bacterium. The master equation accounting for a random switching rate λ of the direction of a rotary motor rotation reads

$$\frac{\partial f(\vec{x},\vec{n},t)}{\partial t} = -\frac{\partial(v\vec{n}f(\vec{x},\vec{n},t))}{\partial\vec{x}} - \frac{\partial(\vec{\omega}\times\vec{v}f(\vec{x},\vec{n},t))}{\partial\vec{v}} - \lambda f(\vec{x},\vec{n},t) + \lambda f(\vec{x},-\vec{n},t). \quad (1)$$

Taking into account that $\vec{v}\times\partial/\partial\vec{v} = \partial/\partial\vartheta$, where $\vec{v} = v\vec{n} = v(\cos\vartheta, \sin\vartheta)$, Eq. (1) may be rewritten for the two-dimensional case in a more simple way as follows:

$$\frac{\partial f(\vec{x},\vec{n},t)}{\partial t} = -\frac{\partial(v\vec{n}f(\vec{x},\vec{n},t))}{\partial\vec{x}} - \omega\frac{\partial(f(\vec{x},\vec{n},t))}{\partial\vartheta} - \lambda f(\vec{x},\vec{n},t) + \lambda f(\vec{x},-\vec{n},t). \quad (2)$$

There are several possible ways to analyse the properties of random trajectories of these circle swimmers. A simple approach is based on the derivation from (2) of a closed set of equations for several moments characterizing their distribution $\langle\vec{x}^2\rangle$, $\langle\vec{x}\,\vec{n}\rangle$, $\langle\vec{x}\,d\vec{n}/d\vartheta\rangle$. It reads

$$\frac{d\langle\vec{x}^2\rangle}{dt} = 2v\langle\vec{x}\,\vec{n}\rangle,$$

$$\frac{d\langle\vec{x}\,\vec{n}\rangle}{dt} = v + \omega\left\langle\vec{x}\frac{d\vec{n}}{d\vartheta}\right\rangle - 2\lambda\langle\vec{x}\,\vec{n}\rangle,$$

$$\frac{d}{dt}\left\langle\vec{x}\frac{d\vec{n}}{d\vartheta}\right\rangle = -\omega\langle\vec{x}\,\vec{n}\rangle - 2\lambda\left\langle\vec{x}\frac{d\vec{n}}{d\vartheta}\right\rangle. \quad (3)$$

In order to derive a set of equations (3) the condition of normalization $\int f(\vec{x},\vec{n},t)\,d\vec{x}\,d^2\vec{n} = 1$ and the relation $d^2\vec{n}/d\vartheta^2 = -\vec{n}$ are used. Since the set (3) is closed it is possible to obtain an analytical solution for $\langle\vec{x}^2\rangle$, which characterizes the random process of the particle diffusion.

* Corresponding author.
E-mail address: aceb@tesla.sal.lv (A. Cebers).

0304-8853/$ - see front matter © 2014 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jmmm.2014.01.004

Please cite this article as: R. Taukulis, A. Cebers, Journal of Magnetism and Magnetic Materials (2014), http://dx.doi.org/10.1016/j.jmmm.2014.01.004

The stationary solution for the moments $\langle \vec{x}\,\vec{n}\rangle$, $\left\langle \vec{x}\frac{d\vec{n}}{d\vartheta}\right\rangle$ reads

$$\langle \vec{x}\,\vec{n}\rangle = \frac{2v\lambda}{(2\lambda)^2+\omega^2}, \tag{4}$$

$$\left\langle \vec{x}\frac{d\vec{n}}{d\vartheta}\right\rangle = -\frac{v\omega}{(2\lambda)^2+\omega^2}. \tag{5}$$

Relations (4) and (5), as a result, give

$$\frac{d\langle \vec{x}^{\,2}\rangle}{dt} = 4\frac{v^2\lambda}{(2\lambda)^2+\omega^2}. \tag{6}$$

Taking into account 2D character of particle motion, absence of cross-correlation $\langle xy\rangle$ in the stationary case and $\langle x^2\rangle = \langle y^2\rangle$ due to the symmetry, for the diffusion coefficient we have

$$D = \frac{v^2\lambda}{(2\lambda)^2+\omega^2}. \tag{7}$$

In the limit $\omega\to 0$ we have

$$D = \frac{v^2}{4\lambda}. \tag{8}$$

Introducing as a characteristic time of tumbling $\tau_s = \lambda^{-1}$ relation (8) reads

$$D = \frac{v^2\tau_s}{4}. \tag{9}$$

It coincides with the expression for the diffusion coefficient of tumbling bacterium [9] with $\langle \cos\,\delta\rangle = -1$, where δ is the angle by which the bacterium changes the direction of motion after a tumbling event. Since in our case direction of the bacterium motion changes to the opposite we have $\langle \cos\,\delta\rangle = -1$. In the limit of high frequency for the diffusion coefficient we have $D = r_0^2/\tau_s$, where r_0 is the radius of a circular trajectory, which coincides with the expression of the diffusion coefficient in the elementary theory of Brownian motion [10].

2. Circle swimmer and thermal noise

We should remark that besides the diffusion due to random switching of the rotary motor of the bacterium there is also thermal noise due to fluctuating forces and torques. This problem is considered in [1,2]. In [1,2] calculation is carried out by using time-correlation functions. We will illustrate further that the diffusion coefficients of torqued swimmers may be calculated in a similar fashion as above on the basis of the Fokker–Planck equation for the joint distribution function of the position and orientation of the swimmer $f(\vec{r},\vec{n},t)$

$$\frac{\partial f}{\partial t} = -\frac{\partial(v\vec{n}f)}{\partial\vec{x}} - \vec{\omega}\vec{K}_{\vec{n}}f + D_R\vec{K}^{2}_{\vec{n}}f + D\frac{\partial^2 f}{\partial\vec{x}^{\,2}} \tag{10}$$

where $\vec{K}_{\vec{n}} = \vec{n}\times\partial/\partial\vec{n}$ is the operator of infinitesimal rotations and D and D_R are the translational and rotational diffusion coefficients respectively. In 3D case the following closed set of equations for the moments may be obtained

$$\frac{d\langle \vec{x}^{\,2}\rangle}{dt} = 2v\langle \vec{x}\,\vec{n}\rangle + 6D,$$
$$\frac{\langle x_z^2\rangle}{dt} = 2D + 2v\langle x_z n_z\rangle,$$
$$\frac{d\langle \vec{x}\,\vec{n}\rangle}{dt} = v + \omega\langle(\vec{n}\times\vec{x})_z\rangle - 2D_R\langle \vec{x}\,\vec{n}\rangle,$$
$$\frac{d\langle x_z n_z\rangle}{dt} = v\langle n_z^2\rangle - 2D_R\langle x_z n_z\rangle,$$
$$\frac{d\langle n_z^2\rangle}{dt} = 2D_R - 6D_R\langle n_z^2\rangle,$$
$$\frac{d\langle(\vec{n}\times\vec{x})_z\rangle}{dt} = -2D_R\langle(\vec{n}\times\vec{x})_z\rangle - \omega\langle \vec{x}\,\vec{n}\rangle + \omega\langle x_z n_z\rangle. \tag{11}$$

In the stationary case we have

$$\langle n_z^2\rangle = \tfrac{1}{3},$$
$$\langle n_z x_z\rangle = \frac{v}{6D_R},$$
$$\langle \vec{x}\,\vec{n}\rangle = \frac{v(1+(\omega/(2D_R))^2/3}{\omega^2/(2D_R)} + 2D_R,$$
$$\frac{d\langle x_z^2\rangle}{dt} = 2D + \frac{v^2}{3D_R},$$
$$\frac{d\langle \vec{x}_\perp^{\,2}\rangle}{dt} = 4D + \frac{2v^2}{3D_R(1+(\omega/(2D_R))^2}. \tag{12}$$

This gives, in agreement with [2],

$$D_\parallel = D + \frac{v^2}{6D_R} \tag{13}$$

and

$$D_\perp = D + \frac{v^2}{6D_R(1+(\omega/(2D_R))^2}. \tag{14}$$

Similar to (14) relation in 2D case, up to our knowledge, was for the first time obtained in [1].

We see that the frequency dependence of the diffusion coefficient is similar to the one for the magnetotactic bacterium under the action of the Poisson like noise. The role of the switching rate λ is determined in this case by the rotary diffusion coefficient of the swimmer D_R. Taking for the rotational drag coefficient of the bacterium the value 2.4×10^{-12} erg s [6] we may estimate the characteristic switching rate due to the rotational diffusion as $0.017\ s^{-1}$, which is much less than expected due to the switching of the rotary motors of a bacterium. This shows that random trajectories observed in [6] of bacteria in a rotating field are due to the random switching of rotary motors and not due to the random thermal noise. Another evidence for this comes from direct comparison of the trajectory observed in the experiment (see [6, Fig. 11]) and trajectory obtained numerically as described in [7] and shown in Fig. 1. Parameters used in generating the trajectory

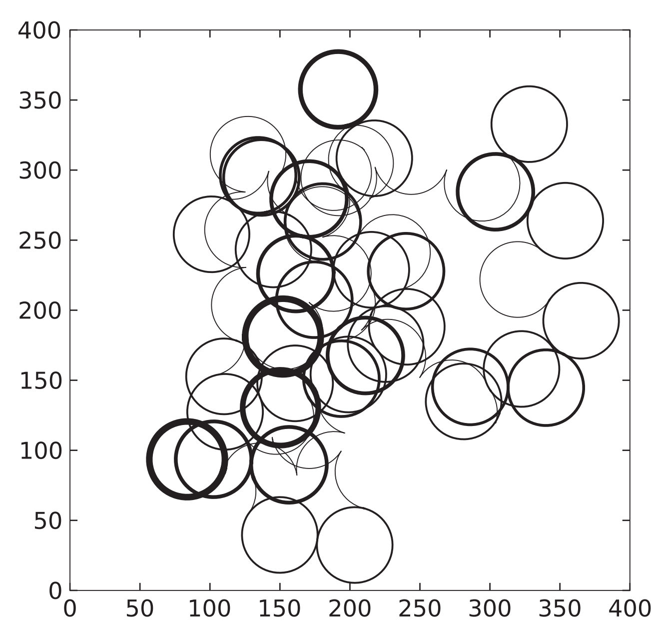

Fig. 1. Generated random trajectory of the bacterium. Line thickness is proportional to the number of full turns before switching of the swimming direction. Coordinates are shown in pixels. 18 pixels = 3.2 μm. See [6, Fig. 11] for a qualitative and quantitative comparison.