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Yield shear stress model of magnetorheological fluids based on exponential distribution

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ABSTRACT

The magnetic chain model that considers the interaction between particles and the external magnetic field in a magnetorheological fluid has been widely accepted. Based on the chain model, a yield shear stress model of magnetorheological fluids was proposed by introducing the exponential distribution to describe the distribution of angles between the direction of magnetic field and the chain formed by magnetic particles. The main influencing factors were considered in the model, such as magnetic flux density, intensity of magnetic field, particle size, volume fraction of particles, the angle of magnetic chain, and so on. The effect of magnetic flux density on the yield shear stress was discussed. The yield stress of aqueous Fe_3O_4 magnetreological fluids with volume fraction of 7.6% and 16.2% were measured by a device designed by ourselves. The results indicate that the proposed model can be used for calculation of yield shear stress with acceptable errors.

1. Introduction

Yield stress is the shear stress to overcome the external force exerted by external magnetic field on magnetorheological fluids when it changes the state from solid-like into liquid. When the applied shear stress is less than its yield stress, the magnetorheological fluid can resist the shear deformation, showing solid state. The deformation increases with the applied shear stress. Once the applied shear stress is larger than its yield stress, the magnetorheological fluid will become fluid [1]. The yield stress is an important performance parameter of the magnetorheological fluid. The measurement and calculation of yield stress of magnetorheological fluid are very important for its application.

Magnetorheological fluid is a functional fluid consisting of carrier liquid and a large number of tiny magnetic particles. Experimental observations show that, in the presence of an external magnetic field, the magnetic particles in the magnetorheological fluid will form chain structure from the original disordered structure [2–4].

The chain model was put forward by Ginder [5]. The chain model was used to calculate the yield shear stress by considering the interaction between particles in a chain. Since then, various models have been established [5–8]. For example, in consideration of magnetic saturation process and nonlinear magnetization process of particles, Jin [6] established a body-centered cubic structure model by using 3D finite element unit. Jang [7] analyzed the shear characteristics of magnetorheological fluid between two infinite plates, studying the interaction between particles in a cube formed by many chains. Dang [8] applied the Maxwell–Boltzmann distribution function to describe the distribution of chains, and modified the dipole model of the magnetorheological elastomers. Li [9] described the angles between the chains of dipoles and the external magnetic field by the normal distribution function, analyzing the effects of various factors on the yield shear stress, and discussing ways for improving the yield shear stress of magnetorheological fluids.

Although there are many ways to calculate the yield shear stress of magnetorheological fluids, it seems that no formula has been widely recognized. Based on the chain model, this paper tried to apply the exponential distribution function to describe the distribution of angles between the direction of magnetic field and the chain formed by magnetic particles. The model was used to calculate the yield shear stress and investigated the effect of magnetic flux density. In order to verify the model established, we designed a device for measurement of the yield shear stress. The yield stress of aqueous Fe_3O_4 magnetreological fluids was measured.

2. Yield shear stress model

The magnetorheological fluids between two plates were studied. A uniform external magnetic field was applied. Lots of experimental results have indicated that the magnetic particles

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in the magnetorheological fluids will be polarized to form magnetic chains in a magnetic field. Exerted by a shear stress, the chain will turn a tiny angle θ, as shown in Fig. 1. If the volume fraction is not very high, the distances between chains will be relatively large, so that the interaction between chains can be neglected when the magnetic flux density is not very large. In the following analysis, only the interactions between particles on the same chain were considered.

Magnetic particles are supposed to be uniform, with a radius r. The distance between two adjacent particles is denoted as δ. According to Coulomb's law, it is easy to get the force between any two particles

$$F_k = \frac{1}{4\pi\mu_0\delta_k^2}\left(\frac{2}{3}\pi r^2\chi B\right)^2 = \frac{\pi r^4\chi^2B^2}{9\mu_0(k\delta)^2} \quad (1)$$

where, r is the radius of magnetic particle; χ is the magnetic susceptibility of magnetic particle; B is the magnetic flux density; μ_0 is the Permeability of vacuum; k is the number of magnetic particles on one chain; δ is the distance between two particles. δ_k is the distance between the two particles that are not adjacent (as shown in Fig. 1).

If the number of particles on one chain is n, then the force of all other particles exerted on the first particle will be

$$F = \sum_{k=1}^{n-1}\frac{\pi r^4\chi^2B^2}{9\mu_0(k\delta)^2} \quad (2)$$

If the chain turns a tiny angle θ, then the distance between particles will become

$$\delta_k = \frac{k\delta}{\cos\,\theta} \quad (3)$$

The horizontal component of force will be

$$F_\tau = \sum_{k=1}^{n-1}\frac{1}{k^2}\times\frac{\pi r^4\chi^2B^2}{9\mu_0\delta^2}\cos^2\theta\ \sin\,\theta \quad (4)$$

In consideration that there are lots of particles on a chain, then $\sum_{k=1}^{n-1}(1/k^2)\approx(\pi^2/6)$. The shear force on one chain is

$$F_\tau = \frac{\pi^3r^4\chi^2B^2}{54\mu_0\delta^2}\cos^2\theta\ \sin\,\theta \quad (5)$$

If the volume fraction of magnetic particles in the magnetorheological fluid is φ, then the number of chains within unit area is

$$m = \frac{3\varphi\delta}{4\pi r^3} \quad (6)$$

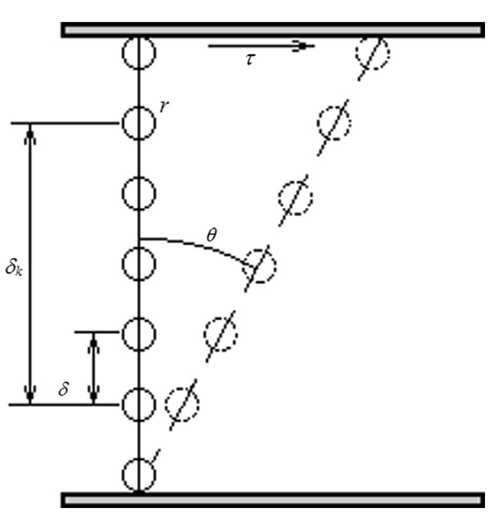

Fig. 1. Stretch of a single chain.

Finally, we obtain the yield shear stress

$$\tau = m\tau' = \frac{\varphi\pi^2r\chi^2B^2}{72\mu_0\delta}\cos^2\theta\ \sin\,\theta \quad (7)$$

where, τ' is the shear stress contributed by one chain.

Although there are factors that affect the forming of chains, the most important one is the magnetic flux density B. In the ideal case, all the chains will be formed parallel to the direction of the external magnetic field, that is, $\theta=0$. However, due to various factors, there must be a few chains which will have an angle with the direction of the external magnetic field. Theoretically, the angle θ could be from 0 to $\pi/2$. Because the external magnetic field will force the chains to be along its direction, it can be assumed that most of the chains will have a very small angle. That is to say, most of the angle θ will be near zero. The distribution of the angles is very similar to an exponential distribution. Therefore, the distribution of the angles can be well described with the exponential distribution.

$$p(\theta) = \lambda e^{-\lambda\theta} \quad (8)$$

where, λ is the rate parameter, denoted as $\lambda=f(B)$. Here, only the effect of magnetic flux density is considered.

The yield shear stress can be expressed as

$$\tau = \int_0^{\pi/2}\frac{\varphi\pi^2r\chi^2B^2}{72\mu_0\delta}\cos^2\theta\ \sin\,\theta\times\lambda\times e^{-\lambda\theta}d\theta \quad (9)$$

Integrating the above equation, we have

$$\tau = \frac{\varphi\pi^2r\chi^2B^2}{72\mu_0\delta}\lambda\left\{\frac{1}{3}+\frac{3\lambda\times e^{-\lambda\pi/2}-\lambda^2}{12(\lambda^2+9)}-\frac{\lambda\times e^{-\lambda\pi/2}+\lambda^2}{4(\lambda^2+1)}\right\} \quad (10)$$

Eq. (10) could be used to predict the yield shear stress of magnetreological fluids if appropriate rate parameters are chosen.

3. Measurement and calculation of yield shear stress

A device with parallel discs was designed for measuring the yield stress of magnetreological fluids, as shown in Fig. 2. Two parallel discs are set in a magnetic field. The diameters of the two discs are 90 mm. The spacing between them is 1 mm. The two discs are inside a container that is filled with magnetreological fluids. A uniform magnetic field is applied along the vertical direction of the discs.

The lower disc is kept stationary, while the upper disc is driven by an external torque. When the external shear stress is lower than the yield stress, the upper disc will not be moved. And the magnetreological fluids between the discs show solid state due to the external magnetic field. When the external shear stress reaches the yield stress, the upper disc begins to rotate. At this moment, the torque transducer will record the value which can be used to calculate the yield stress.

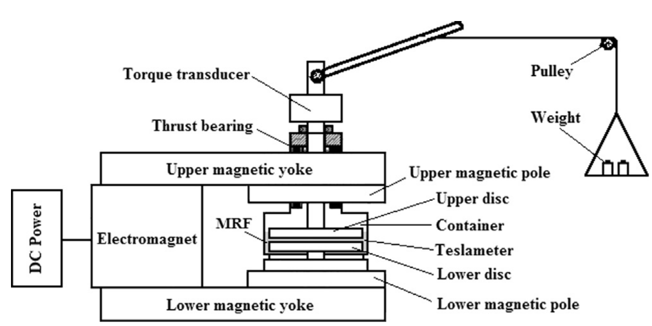

Fig. 2. Experimental apparatus for measuring yield stress of magnetreological fluids.