ARTICLE IN PRESS

Ain Shams Engineering Journal (2014) **xxx**, xxx–xxx

Ain Shams University

Ain Shams Engineering Journal

www.elsevier.com/locate/asej
www.sciencedirect.com

CIVIL ENGINEERING

Vector machine techniques for modeling of seismic liquefaction data

Pijush Samui *

Centre for Disaster Mitigation and Management, VIT University, Vellore 632014, India

Received 10 September 2013; revised 3 December 2013; accepted 24 December 2013

KEYWORDS

Support Vector Machine;
Least Square Support Vector Machine;
Relevance Vector Machine;
Liquefaction;
Probability

Abstract This article employs three soft computing techniques, Support Vector Machine (SVM); Least Square Support Vector Machine (LSSVM) and Relevance Vector Machine (RVM), for prediction of liquefaction susceptibility of soil. SVM and LSSVM are based on the structural risk minimization (SRM) principle which seeks to minimize an upper bound of the generalization error consisting of the sum of the training error and a confidence interval. RVM is a sparse Bayesian kernel machine. SVM, LSSVM and RVM have been used as classification tools. The developed SVM, LSSVM and RVM give equations for prediction of liquefaction susceptibility of soil. A comparative study has been carried out between the developed SVM, LSSVM and RVM models. The results from this article indicate that the developed SVM gives the best performance for prediction of liquefaction susceptibility of soil.

1. Introduction

There is a lot of engineering problems that require the analysis of uncertain and imprecise information. Generally, the development of proper model to explain past behaviors or predict future ones is a difficult task due to incomplete understanding of the problem. Soft computing technique is generally used to solve this type of problem. This technique is developed by Zadeh Iizuka [1]. The most commonly used soft computing technique is Artificial Neural Network (ANN). ANN has been used to solve different problems in engineering [2–6]. However, ANN has the following limitations.

- Unlike other statistical models, ANN does not provide information about the relative importance of the various parameters [7].
- The knowledge acquired during the training of the model is stored in an implicit manner and hence it is very difficult to come up with reasonable interpretation of the overall structure of the network [8].
- In addition, ANN has some inherent drawbacks such as slow convergence speed, less generalizing performance, arriving at local minimum and over-fitting problems.

This article adopts three soft computing techniques {Support Vector Machine (SVM), Least Square Support Vector Machine (LSSVM) and Relevance Vector Machine (RVM)} for prediction of liquefactions susceptibility of soil. Geotechnical

* Tel.: +91 416 2202281; fax: +91 416 2243092.
E-mail address: pijush.phd@gmail.com.

Peer review under responsibility of Ain Shams University.

Production and hosting by Elsevier

2090-4479 © 2014 Production and hosting by Elsevier B.V. on behalf of Ain Shams University.
http://dx.doi.org/10.1016/j.asej.2013.12.004

engineers use the different soft computing techniques for prediction of seismic liquefaction potential of soil [9–13]. The database has taken from the work of Hanna et al. [14]. The dataset contains information about depth of the soil layer (z), corrected standard penetration blow numbers ($N_{1,60}$), percent finest content less than 75 μm ($F \leqslant 75$ μm, %), depth of ground water table (d_w), total and effective overburden stresses ($\sigma_{vo}, \sigma'_{vo}$), threshold acceleration (a_t), cyclic stress ratio (τ_{av}/σ'_{v0}), shear wave velocity (V_s), internal friction angle of soil (ϕ'), earthquake magnitude (M_w), maximum horizontal acceleration at ground surface (a_{max}) and status of soil (status of soil means the condition of soil after earthquake). SVM is a new soft computing technique introduced by Vapnik [15]. There are lots of applications of SVM in engineering [16–20,11–15]. LSSVM is a modified version of SVM [21]. Researchers have successfully used LSSVM for solving different problems [22–26]. RVM was introduced by Tipping [27]. The application of RVM is demonstrated in various literatures [16,28–31]. This article gives equations for prediction of liquefaction susceptibility of soil based on the developed SVM, LSSVM and RVM models. A comparative study has been presented between the developed SVM, LSSVM and RVM models.

2. Details of SVM

SVM was developed based on Structural Risk Minimization Principle [15]. Let us consider the following training dataset (D)

$$D = (x_1, y_1), (x_2, y_2), \ldots, (x_l, y_l), x_i \in R^N \text{ and } y_i \in \{+1, -1\} \quad (1)$$

where x is input, R^N is N-dimensional vector space, and y is output.

In this article, a value of -1 is assigned to the liquefied sites while a value of $+1$ is assigned to the non-liquefied sites so as to make this a two-class classification problem. This study uses z, $N_{1,60}$, $F \leqslant 75$ μm, $d_w, \sigma_{vo}, \sigma'_{vo}, a_t, \tau_{av}/\sigma'_{v0}$, V_s, ϕ', M_w, and a_{max} as input variables. So, $x = [z, N_{1,60}, F \leqslant 75\mu m, d_w, \sigma_{v0}, \sigma'_{v0}, a_t, \tau_{av}/\sigma'_{vo}, V_s, \phi', M_w, a_{\max}]$.

SVM uses the following form for prediction of y.

$$y = \text{sign}(w.\phi(x) + b) \quad (2)$$

$\phi(x)$ represents a high-dimensional feature space which is nonlinearly mapped from the input space x, w is weight and b is bias. The following optimization problem has been used to determine the value of w and b [15].

$$\text{Minimize} : \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{l}\xi_i$$

$$\text{Subjected to} : y_i(w.x_i + b) \geqslant 1 - \xi_i \quad (3)$$

The constant $0 < C < \infty$, a parameter defines the trade-off between the number of misclassification in the training data and the maximization of margin and ξ_i is called slack variable. This optimization problem (4) is solved by Lagrangian Multipliers [15] and its solution is given by,

$$y = sign\left(\sum_{i=1}^{l}\alpha_i y_i K(x_i, x) + b\right) \quad (4)$$

where α_i is Lagrange multipliers and $K(x_i,x)$ is kernel function.

This article uses the above SVM for prediction of liquefaction susceptibility of soil. To develop SVM, the data have been divided into the following two groups:

Training Dataset: This is required to construct the SVM model. This article uses 434 datasets out of 620 as training dataset.

Testing Dataset: This is used to verify the developed SVM. The remaining 185 datasets have used a testing dataset.

Polynomial function ($K(x_i, x) = \{(x_i.x) + 1\}^d$, d = degree of polynomial) has been used as a kernel function. Input variables have been normalized between 0 and 1. The program of SVM has been constructed by MATLAB.

3. Details of LSSVM

This section will describe a brief introduction of LSSVM. The details of LSSVM have been given by Suykens and Vandewalle [21]. The main difference between SVM and LSSVM is that LSSVM uses a set of linear equations for training while SVM uses a quadratic optimization problem [31].

Let us consider the following training dataset (D)

$$D = (x_1, y_1), (x_2, y_2), \ldots, (x_l, y_l), x_i \in R^N \text{ and } y_i \in \{+1, -1\} \quad (5)$$

where x is input, R^N is N-dimensional vector space, and y is output.

In LSSVM, a value of -1 is assigned to the liquefied sites while a value of $+1$ is assigned to the non-liquefied sites so as to make this a two-class classification problem. This study uses z, $N_{1,60}$, $F \leqslant 75$ μm, $d_w, \sigma_{vo}, \sigma'_{vo}, a_t, \tau_{av}/\sigma'_{v0}$, V_s, ϕ', M_w, and a_{max} as input variables. So, $x = [z, N_{1,60}, F \leqslant 75\mu\text{m}, d_w, \sigma_{v0}, \sigma'_{v0}, a_t, \tau_{av}/\sigma'_{vo}, V_s, \phi', M_w, a_{\max}]$.

LSSVM uses the following equation for prediction of y.

$$y = sign[w^T\phi(x) + b] \quad (6)$$

$\phi(x)$ represents a high-dimensional feature space which is nonlinearly mapped from the input space x, w is weight and b is bias.

LSSVM adopts the following optimization problem for determination of w and b.

$$\text{Min} : \frac{1}{2}w^T w + \frac{\gamma}{2}\sum_{i=1}^{l}e_i^2$$

$$\text{Subject to} : e_i = y_i - (w^T\varphi(x_i) + b), i = 1, \ldots, l \quad (7)$$

This optimization problem (4) is solved by Lagrangian Multipliers [21], and its solution is given by

$$y = sign\left(\sum_{i=1}^{l}\alpha_i y_i K(x_i, x) + b\right) \quad (8)$$

where α_i is Lagrange multipliers and $K(x_i.x)$ is kernel function. This study adopts radial basis function ($K(x_i, x) = \exp\left\{-\frac{(x_i - x)^T(x_i - x)}{2\sigma^2}\right\}$ where σ is width of radial basis function) as kernel function.

It should be noted that for model calibration and verification using LSSVM, the same training data sets, testing data sets and normalization technique previously used for the SVM modeling are utilized and LSSVM is implemented using the MATLAB software.