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Transient free convection flow past an accelerated vertical cylinder in a rotating fluid

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Abstract The flow of a viscous incompressible fluid past an accelerated vertical circular cylinder in a rotating fluid is analyzed in this study. The cylinder starts impulsively from rest with uniform acceleration in its own plane relative to the rotating fluid. The closed form solutions of the governing boundary layer equations in non-dimensional form are obtained in terms of Bessel functions and modified Bessel functions by Laplace transform technique. Numerical values of the axial velocity, transverse velocity and temperature profiles are obtained for various physical parameters and presented in graphs. Skin friction and Nusselt number are also discussed graphically.

1. Introduction

The rotating fluid flows plays a vital role in industry, namely, in geophysical, astrophysical and cosmic fluid dynamics. Many researchers have studied the problem of flow past rotating fluid with different physical aspects. Deka et al. [1] studied the flow of viscous incompressible rotating fluid induced by a uniformly accelerated plate. They observed that in presence of rotation, the velocity profiles for varying times are not similar in contrast to the velocity profiles which are similar in absence of rotation.

Xu et al. [2] presented experimentally the flow of a homogeneous, incompressible, rotating fluid past a vertical circular cylinder oscillating laterally in a uniform free stream. The numerical simulation of the two-dimensional incompressible unsteady Navier–Stokes equations for streaming flow past a rotating circular cylinder was studied by Padrino and Joseph [3]. Mittal and Kumar [4] initiated the flow past a spinning circular cylinder placed in a uniform stream. A stabilized finite element method is utilized to solve the incompressible Navier-Stokes equations in the primitive variables formulation. The study of the dynamics of the spin-up time of an incompressible viscous rotating fluid was illustrated by Greenspan and Howard [5]. Das et al. [6] presented an analytical study for the flow of a viscous incompressible fluid due to an oscillating plate in a rotating system. They observed that for large time the transient solution tends to zero, whereas the steady state solution does not exist when the rotation parameter is

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Nomenclature

α	thermal diffusivity	Pr	Prandtl number
β	volumetric coefficient of thermal expansion	r	radial coordinate
c	acceleration of the cylinder	R	dimensionless radial coordinate
g	acceleration due to gravity	T'	fluid temperature
Gr	Grashof number	T	non-dimensional fluid temperature
J_0	Bessel function of first kind and order zero	t'	time
J_1	Bessel function of first kind and order one	t	dimensionless time
Y_0	Bessel function of second kind and order zero	u	axial component of velocity
Y_1	Bessel function of second kind and order one	v	transverse component of velocity
K_0	modified Bessel function of second kind and order zero	U	dimensionless axial velocity
		V	dimensionless transverse velocity
K_1	modified Bessel function of second kind and order one	ω	rotation of the cylinder
		Ω	dimensionless rotation parameter
ν	kinematic viscosity		

equal to the frequency parameter. Hayat et al. [7] presented an analysis of an electrically conducting viscous fluid over a porous plate in a rotating system. Recently, Deka and Paul [8] studied the flow past an impulsively started horizontal cylinder in a rotating fluid. They have demonstrated that the fluid velocity as well as skin friction approach to steady state at larger time.

However, the flow past vertical cylinders in a rotating fluid has never been considered in the literature as far as analytical treatment is concerned. This may be due to the complicated mathematical calculation involved in the solution of such problems. The aim of the present paper is to investigate the boundary layer flow past an infinite vertical cylinder in a rotating fluid. It would be of interest to see how the flow past a vertical cylinder in a rotating fluid gets modified. This is because of the coriolis force due to the rotation manifests itself in changing the pattern of flows. We have discussed the subsequent flow when the cylinder started impulsively from rest (relative to the rotating fluid) moves with uniform acceleration in its own plane. The closed form solutions of the boundary layer governing partial differential equations are obtained in terms of Bessel functions and modified Bessel functions by usual Laplace transform technique. The effects of various significant parameters on the flow and heat transfer characteristics are presented in graphical form. The mathematical formulation of the problem is presented in Section 2, followed by the analytic solution procedure in Section 3. Results and discussion are presented in Section 4 and conclusions in Section 5.

2. Mathematical formulation

Consider the unsteady free convection flow of an incompressible viscous fluid past an infinite vertical cylinder of radius r_0, the x-axis is taken vertically upward along the axis of the cylinder and the radial coordinate r is taken normal to it. Initially at $t' \leqslant 0$, it is assumed that the cylinder is at rest and the cylinder and fluid are at the same temperature T'_∞. At $t' > 0$, the temperature of the cylinder raised to constant temperature T'_w and the cylinder starts with uniform acceleration c in its own plane relative to the fluid which is rotating in unison with uniform angular velocity ω about the axis of the cylinder. It is also assumed that all the fluid properties are constant except for the density in the buoyancy term, which is given by the usual Boussinesq's approximation. Under these assumptions the governing boundary layer equations are given by,

$$\frac{\partial u}{\partial t'} = \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + 2\omega \mathrm{v} + g\beta\left(T' - T'_\infty\right)\cos\omega t' \tag{1}$$

$$\frac{\partial \mathrm{v}}{\partial t'} = \nu\left(\frac{\partial^2 \mathrm{v}}{\partial r^2} + \frac{1}{r}\frac{\partial \mathrm{v}}{\partial r}\right) - 2\omega u - g\beta\left(T' - T'_\infty\right)\sin\omega t' \tag{2}$$

$$\frac{\partial T'}{\partial t'} = \alpha\left(\frac{\partial^2 T'}{\partial r^2} + \frac{1}{r}\frac{\partial T'}{\partial r}\right) \tag{3}$$

with initial and boundary conditions,

$$\left.\begin{array}{llll} t' \leqslant 0: & u = 0, \quad \mathrm{v} = 0, & T' = T'_\infty & \text{for all } r \\ t' > 0: & u = ct', \quad \mathrm{v} = 0, & T' = T'_w & \text{at } r = r_0 \\ & u \to 0, \quad \mathrm{v} \to 0, & T' \to T'_\infty & \text{as } r \to \infty \end{array}\right\} \tag{4}$$

Introducing the non-dimensional quantities,

$$\left.\begin{array}{l} R = \frac{r}{r_0}, \quad t = \frac{t'\nu}{r_0^2}, \quad U = \frac{u\nu}{r_0^2 c}, \quad V = \frac{\mathrm{v}\nu}{r_0^2 c}, \quad T = \frac{T'-T'_\infty}{T'_w-T'_\infty}, \\ \Omega = \frac{r_0^2\omega}{\nu}, \quad \Pr = \frac{\nu}{\alpha}, \quad Gr = \frac{g\beta\left(T'_w-T'_\infty\right)}{c} \end{array}\right\} \tag{5}$$

the governing Eqs. (1)–(3) reduce to,

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial R^2} + \frac{1}{R}\frac{\partial U}{\partial R} + 2\Omega V + GrT\cos\Omega t \tag{6}$$

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial R^2} + \frac{1}{R}\frac{\partial V}{\partial R} - 2\Omega U - GrT\sin\Omega t \tag{7}$$

$$\frac{\partial T}{\partial t} = \frac{1}{Pr}\left(\frac{\partial^2 T}{\partial R^2} + \frac{1}{R}\frac{\partial T}{\partial R}\right) \tag{8}$$

with the corresponding initial and boundary conditions,

$$\left.\begin{array}{llll} t \leqslant 0: & U = 0, \quad V = 0, & T = 0 & \text{for all } R \\ t > 0: & U = t, \quad V = 0, & T = 1 & \text{at } R = 1 \\ & U \to 0, \quad V \to 0, & T \to 0 & \text{as } R \to \infty \end{array}\right\} \tag{9}$$

Eqs. (6) and (7) can be combined into a single equation,

$$\frac{\partial W}{\partial t} = \frac{\partial^2 W}{\partial R^2} + \frac{1}{R}\frac{\partial W}{\partial R} - 2i\Omega W + GrTe^{-i\Omega t} \tag{10}$$

where $W(R,t) = U(R,t) + iV(R,t)$ with