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Numerical and analytical treatment on peristaltic flow of Williamson fluid in the occurrence of induced magnetic field

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ABSTRACT

In this paper the effects of induced magnetic field on the peristaltic transport of a Williamson fluid model in an asymmetric channel has been investigated. The problem is simplified by using long wave length and low Reynolds number approximations. The perturbation and numerical solutions have been presented. The expressions for pressure rise, pressure gradient, stream function, magnetic force function, current density distribution have been computed. The results of pertinent parameters have been discussed graphically. The trapping phenomena for different wave forms have been also discussed.

1. Introduction

Since the pioneering work done by Latham [1], considerable attention has been given to the study of peristaltic flows of both Newtonian and non-Newtonian fluids with different flow geometries because of their importance in many engineering and Biomedical applications. In biological systems it is involved in urine transport from kidney to bladder, swallowing food through esophagus, chyme motion in the gastrointestinal tract, vasomotion of small blood vessels and movement of spermatozoa in the human reproductive tract. There are many engineering processes as well in which peristaltic pumps are used to handle a wide range of fluids particularly in chemical and pharmaceutical industries. It is used in sanitary fluid transport, blood pumps in heart lungs machine and transport of corrosive fluids where the contact of the fluid with the machinery parts are prohibited. Peristaltic flows of Newtonian and non-Newtonian fluids with different physical geometries and wave shapes have been studied by number of authors. To mention a few, Nadeem and Akbar [2] have examined the effects of temperature dependent viscosity on peristaltic flow of a Jeffrey six constant fluid in a non-uniform tube. Lozano and Sen [3] have highlighted the stream lines patterns and their local and global bifurcation in a two dimensional planner and axisymmetric peristaltic flow of a Newtonian fluid. They [3] discussed three bifurcation regions and verify their results with the experimental data. The peristaltic flow of a couple stress fluid in an annulus have been studied by Mekheimer and Abd-elmaboud [4]. Nadeem and Akram [5–8] have discussed the peristaltic flows of Newtonian and non-Newtonian fluid in symmetric and asymmetric channels. In few other papers, Nadeem and Akbar [9–12] have discussed the peristaltic flows in cylindrical geometry with different wave forms. A new numerical solution for MHD peristaltic flow of a bio-fluid with variable viscosity in a circular cylindrical tube via Adomian decomposition method has been examined by Ebaid [13]. Some more useful papers on this subject are cited in the Refs. [14–25] In view of amount of work done on peristaltic flows it becomes interesting to investigate the effects of induced magnetic field on the peristaltic flow of Williamson fluid model in symmetric and asymmetric channel. The modeled nonlinear equations of Williamson model for two dimensional peristaltic flow are simplified using the well-known long wave length and low Reynolds number approximations. The reduced equations are then solved analytically and numerically. A comparison of both the solutions is also given. The expressions for pressure rise, velocity, induced magnetic field function and stream lines are discussed through graphs for different physical parameters Tables 1 and 2.

2. Mathematical formulation

Let us consider the peristaltic flow of an incompressible, electrically conducting non-Newtonian fluid (Williamson fluid)

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Nomenclature

U,V	velocity components in the X and Y directions in fixed frame
U,v	velocity components in the x and y directions in wave frame
ρ	constant velocity
p	pressure
σ	electrical conductivity
M	Hartmann number
Re	Reynolds number
Φ	amplitude ratio
a1;b1	amplitude of waves
λ	wave length
c	velocity of propagation
Q	volume flow rate
δ	long wave length
ψ	stream function
S1	Strommer's number (magnetic force number)
Rm	magnetic Reynolds numbers
Φ	magnetic force function
τ	extra stress tensor

in a two dimensional channel of width d_1+d_2. The flow is generated by sinusoidal wave trains propagating with constant speed c along the channel walls. We choose a rectangular coordinate system for the channel with X along the center line of the channel and Y is transverse to it. An external transverse uniform constant magnetic field H_0, induced magnetic field H $(h_X(X,Y,t),H_0+h_Y(X,Y,t),0)$ and the total magnetic field $H^+(h_X(X,Y,t), H_0+h_Y(X,Y,t),0)$ are taken into account.

A schematic diagram of the geometry of the problem under consideration is shown in Fig. (a).

The channel walls are considered to be non-conductive and the geometry of the wall surface is defined as

$$Y=H_1=d_1+a_1\,\mathrm{Cos}\left[\frac{2\pi}{\lambda}(X-ct)\right],$$
$$Y=H_2=-d_2-b_1\,\mathrm{Cos}\left[\frac{2\pi}{\lambda}(X-ct)+\phi\right], \tag{1}$$

where a_1 and b_1 are the amplitudes of the waves, λ is the wave length, d_1+d_2 is the width of the channel, c is the velocity of propagation, t is the time and X is the direction of wave propagation, the phase difference ϕ varies in the range $0\le\phi\le\pi$, $\phi=0$ corresponds to symmetric channel with waves out of phase and $\phi=\pi$, the waves are in phase, further a_1,b_1,d_1,d_2 and ϕ satisfy the condition

$$a_1^2+b_1^2+2a_1b_1\cos\,\phi\le(d_1+d_2)^2.$$

The equations governing the flow are given by

(i). Maxwell's equation

$$\nabla H=0,\ \nabla E=0, \tag{2}$$

$$\nabla\wedge H=\mathbf{J},\mathrm{with}\mathbf{J}=\boldsymbol{\sigma}\{E+\boldsymbol{\mu}_e(V\wedge H)\}, \tag{3}$$

$$\nabla\wedge E=-\boldsymbol{\mu}_e\frac{\partial\mathbf{H}}{\partial\mathbf{t}}. \tag{4}$$

(ii). The continuity equation

$$\nabla V=0. \tag{5}$$

(iii). The equation of motion

$$\rho\left(\frac{\partial\mathbf{V}}{\partial t}+(\mathbf{V}\cdot\nabla)\,\mathbf{V}\right)=div(-\mathrm{pI}+\boldsymbol{\tau})-\nabla\left(\frac{1}{2}\boldsymbol{\mu}_e\left(\mathbf{H}^+\right)^2\right)-\boldsymbol{\mu}_e\left(\mathbf{H}^+\cdot\nabla\right)\mathbf{H}^+, \tag{6}$$

in which the extra stress tensor τ for Williamson fluid in defined by [5]

$$\boldsymbol{\tau}=\mu_0\left[(1-\Gamma\dot{\gamma})^{-1}\right]\dot{\gamma}=\mu_0[(1+\Gamma\dot{\gamma})]\,\dot{\gamma}. \tag{7}$$

With the help of Eqs. (2)–(4), we obtain the induction equation as follows:

$$\frac{\partial\mathbf{H}^+}{\partial t}=\nabla\wedge\left(V\wedge\mathbf{H}^+\right)+\frac{1}{\xi}\nabla^2\mathbf{H}^+, \tag{8}$$

where $\xi=\frac{1}{\sigma\boldsymbol{\mu}_e}$ is the magnetic diffusively.

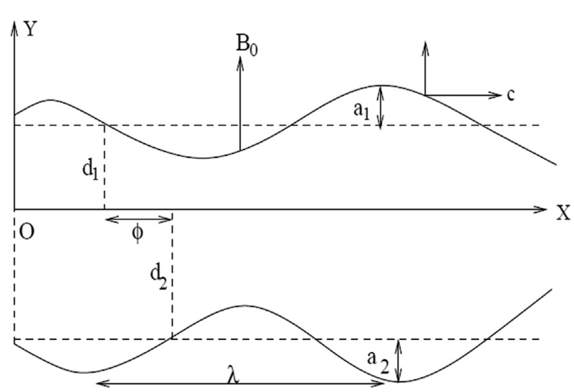

Fig. (a). Geometry of the problem.

Table 1
Shows the comparison of Numerical and Perturbation solution.

y	Perturbation solution	Numerical solution
−1.5	−1	−1
−1.2	−1.75134	−1.726386988
−0.9	−2.32132	−2.273336808
−0.6	−2.71565	−2.649944075
−0.3	−2.93986	−2.863689314
0	−2.99925	−2.920718863
0.3	−2.89899	−2.826056927
0.6	−2.6441	−2.583769801
0.9	−2.23945	−2.197095457
1.2	−1.68984	−1.668547535
1.5	−1	−1

Table 2
Shows the comparison of Numerical and perturbation solution.

Y	Perturbation solution	Numerical solution
−1.5	−1	−1
−1.2	−1.73181	−1.726386988
−0.9	−2.28729	−2.273336808
−0.6	−2.67189	−2.649944075
−0.3	−2.89086	−2.863689314
0	−2.94927	−2.920718863
0.3	−2.85202	−2.826056927
0.6	−2.60387	−2.583769801
0.9	−2.20946	−2.197095457
1.2	−1.67335	−1.668547535
1.5	−1	−1