Journal of Magnetism and Magnetic Materials 343 (2013) 55–59

Contents lists available at SciVerse ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Analytical approximations to the core radius and energy of magnetic vortex in thin ferromagnetic disks

Konstantin L. Metlov *

Donetsk Institute for Physics and Technology NAS, Donetsk 83114, Ukraine

ARTICLE INFO

Article history:
Received 2 February 2013
Received in revised form
22 April 2013
Available online 3 May 2013

Keywords:
Micromagnetics
Magnetic nano-dots
Magnetic vortex

ABSTRACT

The energy of magnetic vortex core and its equilibrium radius in thin circular cylinder were first presented by Usov and Peschany in 1994. Yet, the magnetostatic function, entering the energy expression, is hard to evaluate and approximate. Here, precise and explicit analytical approximations to this function (as well as equilibrium vortex core radius and energy) are derived in terms of elementary functions. Also, several simplifying approximations to the magnetic Hamiltonian and their impact on theoretical stability of magnetic vortex state are discussed.

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1. Introduction

The first topological soliton was discovered as a solution of non-linear field theory equations by Skyrme [1]. It had a form of three-dimensional hedgehog and was named subsequently "skyrmion" in honor of the discoverer. After the landmark work of Belavin and Polyakov [2], topological solitons have crossed the boundary into condensed matter physics. The latter authors discovered much more topological soliton solutions in the infinite Heisenberg ferromagnet, as many as there are rational functions of complex variable, mapping any such function into some equilibrium magnetic structure. Zeros of numerator (possibly with higher multiplicity) of these rational functions correspond to the centers of magnetic vortices, while zeros of denominator to the centers of magnetic anti-vortices. In the model of infinite 2D ferromagnet, considered by Belavin and Polyakov, solitons are absolutely stable. Once magnetic texture with a certain topological charge (number of magnetic vortices) is created, all the structures with different topological charge are separated by an infinite energy barrier. It is worth noting that 7 years earlier essentially the same mathematics of rational functions of complex variable was applied to the problem of magnetic singularities (Bloch points) in 3D ferromagnet by Döring [3], who also found that the exchange energy of ferromagnet around a Bloch point depends only on degrees of numerator and denominator of the corresponding rational function. His energy expression is exactly the same as that of Belavin and Polyakov [2] for 2D ferromagnet. However (probably, because Bloch points, to which model of Ref. [3] applies, are rather exotic objects in magnetism), the paper by Döring is currently much less known and cited.

The model of Belavin and Polyakov became known in particle physics as non-linear O(3) σ model in 3+1 dimensions and reformulated elegantly in terms of functions of complex variable by Woo [4]. Gross found additional family of "meron" solutions to it [5]. Since then, the original Belavin–Polyakov solutions became known as just "solitons". Merons, and all other O(3) σ model solutions besides solitons [4], have infinite energy in unbounded 2-d ferromagnet, but can be realized when the ferromagnet is finite [6].

While these solutions were obtained long ago, the question of their stability has a history of its own. Kosterlitz and Thouless [7] analyzed stability of planar vortices in 2D ferromagnet and came to conclusion that they are unstable and such order could not exist. It is, indeed, true that the energy of Belavin–Polyakov solitons is scale-invariant and their size is, thus, undefined. In real ferromagnets, however, there are various other interactions (not exotic at all), which make the vortices stable. Usov and Peschany, were first to show that dipolar magnetostatic interaction stabilizes magnetic vortex in ferromagnetic cylinder both with respect to core radius change [8] and vortex center displacement [9]. Their results were later fully confirmed experimentally, starting with the direct observation of magnetic vortex core and measurement of its radius [10]. These and the following experiments made single vortex state not only interesting from fundamental point of view, but also an essential component of emerging spintronic devices (such as MRAM elements, based on vortex core polarity [11] or chirality [12] switching, or spin-polarized current magnetic nano-oscillators [13]). It is also prerequisite for study of more complex

* Tel.: +380 62 231 13766.
E-mail address: metlov@fti.dn.ua

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http://dx.doi.org/10.1016/j.jmmm.2013.04.078

multi-vortex magnetic configurations in planar nano-elements of various shapes.

Here, starting from recent (and more general) description of magnetization distributions in finite nano-elements via functions of complex variable [14] the impact of various approximations on vortex stability is reviewed in unified manner and the expression for vortex core radius in circular cylinder [8] is re-derived. It defines the core radius implicitly via an equation and an integral of certain special functions, which is very inconvenient to evaluate and approximate at small cylinder thickness because in this limit it is not analytic and its higher derivatives do not exist. It is, however, possible to introduce small parameters and expand the special functions and the vortex core radius into series, obtaining an explicit analytical approximate (but very precise) expressions, presented at the end.

2. Magnetic vortex in complex variables and its exchange energy

In finite planar nano-elements the equilibrium magnetization configurations can be described via rational functions of complex variable with real coefficients [14] (as opposed to complex coefficients in the case of infinite film [2,4]). The simplest ansatz for magnetic vortex in circular cylinder (of thickness L_Z and radius R) can be written in the complex notation as

$$f(z) = \imath(z-a)/R_V \tag{1}$$

where $z = X + \imath Y$ with X and Y being the Cartesian coordinates in the cylinder's plane (the magnetization distribution is assumed to be independent on out-of-plane coordinate Z), R_V is the vortex core radius and a is the displacement of the vortex from the origin ($a=0$ corresponds to the centered vortex). Let us then define a complex function

$$w(z,\bar{z}) = \begin{cases} f(z) & |f(z)| \le 1 \\ f(z)/|f(z)| & |f(z)| > 1 \end{cases}, \tag{2}$$

where the line over variable denotes complex conjugation. The function w is shown to depend explicitly on both z and $\bar{z}$ because it is, in general, not holomorphic. It consists of two parts: soliton (where it is analytic and $\partial w(z,\bar{z})/\partial\bar{z} = 0$) and meron (where $w\bar{w} = 1$, joined at a line possibly multiply connected if there are several vortices or anti-vortices) $|f| = 1$. The magnetization components, normalized by material's saturation magnetization M_S, are then expressed via stereographic projection as

$$m_x + \imath m_y = \frac{2w(z,\bar{z})}{1 + w(z,\bar{z})\overline{w}(z,\bar{z})} \tag{3}$$

$$m_z = \frac{1 - w(z,\bar{z})\overline{w}(z,\bar{z})}{1 + w(z,\bar{z})\overline{w}(z,\bar{z})}. \tag{4}$$

Being written via the magnetization components in Eqs. (3) and (4), the ansatz in Eq. (1) is exactly equivalent to the one by Usov and Peschany [8] and also belongs to the class of trial functions, considered by Kosterlitz and Thouless [7]. Following the latter work, let us first take into account only the exchange interaction. In complex notation the exchange energy density (omitting the factor $C/2$, where C is the exchange stiffness) can be directly expressed via the function w:

$$\sum_{i=x,y,z} (\vec{\nabla} m_i)^2 = \frac{8}{(1 + w\overline{w})^2} \left(\frac{\partial w}{\partial z}\frac{\partial \overline{w}}{\partial \bar{z}} + \frac{\partial w}{\partial \bar{z}}\frac{\partial \overline{w}}{\partial z} \right), \tag{5}$$

where $\partial/\partial z = (\partial/\partial X - \imath\partial/\partial Y)/2$ and $\partial/\partial\bar{z} = (\partial/\partial X + \imath\partial/\partial Y)/2$. The total exchange energy can be obtained by integrating the density (5) over nano-element's volume. Recalling the Riemann–Green theorem

$$\frac{1}{2\imath}\oint_{\partial\mathcal{D}} u(\zeta,\bar{\zeta})\,\mathrm{d}\zeta = \iint_{\mathcal{D}} \frac{\partial u(z,\bar{z})}{\partial\bar{z}}\,\mathrm{d}X\,\mathrm{d}Y, \tag{6}$$

where u is a complex function of the complex argument (not necessary analytic[1]), it is possible to reduce the area integral over cylinder's face $\mathcal{D}$ for the total exchange energy to a contour integral over its boundary $\partial\mathcal{D}$, provided there is a complex function, whose derivative over $\bar{z}$ yields the exchange energy density (5). Luckily, such function (actually two functions, one for soliton and one for meron part of w) can be easily obtained by direct integration of (5) with w from each of the conditions in (2):

$$u^{\mathrm{S}}(z,\bar{z}) = -\frac{8}{1 + f(z)\bar{f}(\bar{z})}\frac{1}{f(z)}\frac{\partial f}{\partial z}, \tag{7}$$

$$u^{\mathrm{M}}(z,\bar{z}) = \frac{1}{f(z)}\frac{\partial f}{\partial z}\log(f(z)\bar{f}(\bar{z})). \tag{8}$$

Thus, from (6), the total exchange energy inside the soliton is

$$\frac{E^{\mathrm{S}}_{\mathrm{EX}}}{CL_Z/2} = \frac{2}{\imath}\oint_{|f(\zeta)|=1}\frac{1}{f(\zeta)}\frac{\partial f(\zeta)}{\partial\zeta}\,\mathrm{d}\zeta, \tag{9}$$

where the fact that $|f(\zeta)| = 1$ on the integration contour is used and the additional minus sign appears because the original contour of integration has to be walked clockwise. The function under the integral is analytic everywhere except the vortex centers z_i, where $f(z_i) = 0$. Assuming that line $|f(\zeta)| = 1$ does not cross the particle boundary, it is possible to tighten the contours around each topological singularity (vortex or anti-vortex center) and use the residue theorem

$$\frac{E^{\mathrm{S}}_{\mathrm{EX}}}{CL_Z/2} = 4\pi\sum_i \mathrm{Res}\frac{1}{f(z)}\frac{\partial f(z)}{\partial z}\Big|_{z\to z_i}. \tag{10}$$

In particular, for $f(z)$ from Eq. (1) this gives $E^{\mathrm{S}}/(CL_Z/2) = 4\pi$. If there are several vortices inside the particle, the energy will be multiplied by their total number, including multiplicities.

For the meron part, the integration boundary is multiply connected. However, on the inner boundaries (encircling solitons) $|f(\zeta)| = 1$ and $u^{\mathrm{M}}(z,\bar{z}) \sim \log 1 = 0$. Thus, only the integral over the cylinder's outer boundary remains

$$\frac{E^{\mathrm{M}}_{\mathrm{EX}}}{CL_Z/2} = \frac{1}{2\imath}\oint_{\partial\mathcal{D}}\frac{1}{f(\zeta)}\frac{\partial f(\zeta)}{\partial\zeta}\,\log f(\zeta)\bar{f}(\bar{\zeta})\,\mathrm{d}\zeta \tag{11}$$

for $f(z)$ from Eq. (1) and the nano-element, shaped as circular cylinder ($\partial\mathcal{D}$ is $|z| = R$)

$$\frac{E^{\mathrm{M}}_{\mathrm{EX}}}{CL_Z/2} = \int_0^{2\pi}\frac{(1 - a\,\cos(\varphi)/R)\,\log\left(\frac{a^2 - 2aR\,\cos(\varphi) + R^2}{R_V^2}\right)}{2(a^2/R^2 - 2a\,\cos(\varphi)/R + 1)}\,\mathrm{d}\varphi = \pi\,\log\left(1 - \frac{a^2}{R^2}\right) - 2\pi\,\log\left(\frac{R_V}{R}\right), \tag{12}$$

and the total exchange energy $e_{\mathrm{EX}} = (E^{\mathrm{S}} + E^{\mathrm{M}})/(\mu_0\gamma_B M_S^2 \pi R^2 L_Z)$ (in subsequent text all the dimensionless energies, denoted by small letter e with different sub-/superscripts use the same normalization) is

$$e_{\mathrm{EX}} = \frac{L_E^2}{R^2}\left(2 - \log\frac{R_V}{R} + \log\sqrt{1 - \frac{a^2}{R^2}}\right), \tag{13}$$

where $\gamma_B = 4\pi$, $\mu_0 = 1$ in CGS units and $\gamma_B = 1$ in SI [15] and the exchange length[2] $L_E = \sqrt{C/(\mu_0\gamma_B M_S^2)}$. It can be seen immediately

[1] For analytic u the double integral over $\mathcal{D}$ is equal to 0, which is the manifestation of Cauchy theorem.

[2] The other common definition of the exchange length $L_E^{\mathrm{UP}} = \sqrt{C/(\mu_0 M_S^2)}$, used by Usov and Peschany [8] and in many followup works, is, actually, dependent on system of measurement units and makes the formulas for the dimensionless energy and all the derived quantities depend on units too. To avoid this complication the definition $L_E = \sqrt{C/(\mu_0\gamma_B M_S^2)}$ is adopted here, which in CGS units (which

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