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Combining micromagnetism and magnetostatic Maxwell equations for multiscale magnetic simulations ☆

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ABSTRACT

Magnetostatic Maxwell equations and the Landau–Lifshitz–Gilbert (LLG) equation are combined to a multiscale method, which allows to extend the problem size of traditional micromagnetic simulations. By means of magnetostatic Maxwell equations macroscopic regions can be handled in an averaged and stationary sense, whereas the LLG allows to accurately describe domain formation as well as magnetization dynamics in some microscopic subregions. The two regions are coupled by means of their strayfield and the combined system is solved by an optimized time integration scheme.

1. Introduction

Micromagnetic simulations are utilized in a wide range of applications ranging from magnetic storage devices, permanent magnets to spintronic devices. With increasing complexity of the devices more properties have to be included in the simulations in order to predict the functional behavior of the structures accurately. State of the art micromagnetic simulations can handle systems with several millions of unknowns. In order to tackle these large scale problems both (i) new hardware architectures [1,2] as well as (ii) advanced numerical methods are required.

Newly developed numerical methods focus on speeding up the two most time consuming parts in micromagnetic simulations, which are the calculation of the strayfield and the time integration of the LLG equation. Advanced time integrations schemes can be found in Refs. [3–8]. For the calculation of the strayfield advanced FFT algorithms [9,10], fast multipole methods [11,12], nonuniform grid methods [13], FEM/BEM coupling approaches including compression of the boundary matrix [14–16], and tensor grid methods [17,18] have been developed.

Aside from new algorithms solving the micromagnetic model efficiently for systems with many degrees of freedom, it is often possible to choose a simplified physical model to describe at least some parts of the total problem. By this way the number of degrees of freedom can be reduced dramatically without loosing accuracy in regions where it is desired. Within this paper we will utilize the fact that models described by the LLG equation require very fine grained discretization which can lead to impractically large system sizes. We propose using the LLG equation to describe only those regions of the problem where detailed information about the domain structure such as domain walls and vortex structures are required. For the rest of the model a macroscopic description via magnetostatic Maxwell equations is chosen. Since it does not resolve the detailed domain structure it allows to use much coarser discretization.

In contrast to the multiscale method presented in this paper there exist methods which solve combined LLG–Maxwell equations within the whole problem region [19–21]. These methods extend the ordinary LLG model, by allowing to describe eddy currents or other dynamic effects, but they do not address the discretization size constraint and are therefore not suitable for large scale problems.

The structure of the paper is as follows. Section 2 summarizes the methods that are used to individually solve the LLG equations or magnetostatic Maxwell equations respectively. How the two systems can be coupled in an efficient way is described in Section 3. Finally in Section 4 the multiscale algorithm is applied to the simulation of a magnetic giant magnetoresistance (GMR) read head and numerical results and benchmarks are presented.

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2. Fundamentals

For the coupling of micromagnetism and magnetostatic Maxwell equations the full model is divided into two separated regions (see Fig. 1). The LLG equation is used to describe the first region Ω_{llg}, where domain structure, short range interactions or the magnetization dynamics of the magnetic parts is of great interest. The second region Ω_{max} is described by magnetostatic Maxwell equations, which describe the magnetic state in a spatially averaged sense and without dynamics. Since both models contain the external field as a source term, coupling via the strayfield can be achieved in a straightforward way. The strayfield created from the LLG model can be considered as an external field of the Maxwell model and vice versa. An additional region Ω_{coil} allows to define currents in a nonmagnetic medium, which in turn creates the source field for the magnetic model. The solution of the open-boundary problem requires the definition of the boundaries of the LLG-region (Γ_{llg}) as well as of the Maxwell region (Γ_{max}). In the following subsections it is shown how the two subproblems are solved individually.

2.1. LLG

The LLG equation describes how magnetic polarizations $\mathbf{J}$ (with a fixed modulus J_s) evolve in an effective field $\mathbf{H_{eff}}$. It consists of a precessional term as well as a phenomenological damping term

$$\frac{\partial \mathbf{J}}{\partial t} = -\frac{|\gamma|}{1+\alpha^2}\mathbf{J}\times\mathbf{H_{eff}} - \frac{\alpha}{1+\alpha^2}\frac{|\gamma|}{J_s}\mathbf{J}\times\mathbf{J}\times\mathbf{H_{eff}} \tag{1}$$

where α is the Gilbert damping constant, J_s is the saturation polarization and $|\gamma| = \mu_0|\gamma_e| = 2.210175\times 10^5$ m/As is the reduced gyromagnetic ratio (with μ_0 the permeability of the free space and γ_e the gyromagnetic ratio of the electron). The effective field can be split into four contributions as follows:

$$\mathbf{H_{eff}} = \mathbf{H_{ex}} + \mathbf{H_{ani}} + \mathbf{H_{demag}} + \mathbf{H_{ext}}$$
$$= \frac{2A}{J_s^2}\Delta\mathbf{J} - \frac{2}{J_s^2}K_1(\mathbf{J}\cdot\mathbf{a})\mathbf{a} + \mathbf{H_{demag}} + \mathbf{H_{ext}} \tag{2}$$

$\mathbf{H_{ex}}$ describes the short-range exchange interaction parametrized by the exchange constant A. $\mathbf{H_{ani}}$ stands for the magnetocrystalline anisotropy field with the uniaxial anisotropy constant K_1 and the easy axis $\mathbf{a}$. The magnetic strayfield $\mathbf{H_{demag}}$ describes the long-range interaction between the magnetic moments within the magnetic medium. $\mathbf{H_{ext}}$ is the applied field, which can for example be created by an electric coil, or as described later on by a Maxwell model. In addition to the mentioned fields several other contributions are possible, like terms taking into account thermal fluctuations or magneto-elastic interactions.

To calculate the strayfield created by a given magnetization distribution, which is needed for $\mathbf{H_{demag}}$ and also for the interaction between LLG and Maxwell parts, the Fredkin–Koehler method [14] is used. Basically the following equations for the scalar potential u_{llg} are solved for given $\mathbf{J}$:

$$\nabla^2 u_{llg} = \nabla\cdot\mathbf{J} \quad \text{in } \Omega_{llg} \tag{3a}$$

$$\nabla^2 u_{llg} = 0 \quad \text{in } \mathbb{R}^3\backslash\Omega_{llg} \tag{3b}$$

$$[u_{llg}] = 0 \quad \text{on } \Gamma_{llg} \tag{3c}$$

$$\left[\frac{\partial u_{llg}}{\partial \mathbf{n}}\right] = \mathbf{n}\cdot\mathbf{J} \quad \text{on } \Gamma_{llg} \tag{3d}$$

where $[x]$ means the jump of value x at the surface of the LLG region. The strayfield finally reads as $\mathbf{H_{demag}} = -\mu_0^{-1}\nabla u_{llg}$.

A detailed description of how the LLG equation is actually solved as well as a proper preconditioning method to speed up calculations of large problems can be found in [3].

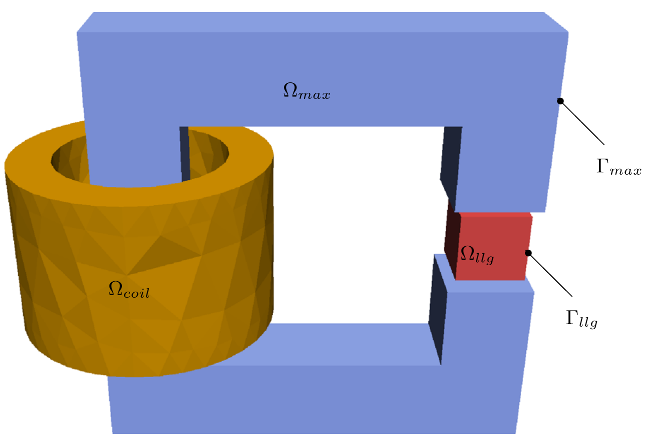

Fig. 1. Example geometry which demonstrates model separation into LLG region Ω_{llg} and Maxwell region Ω_{max} (and in this case in an electric coil region Ω_{coil}). The boundaries of the regions are called Γ_{llg} and Γ_{max} respectively.

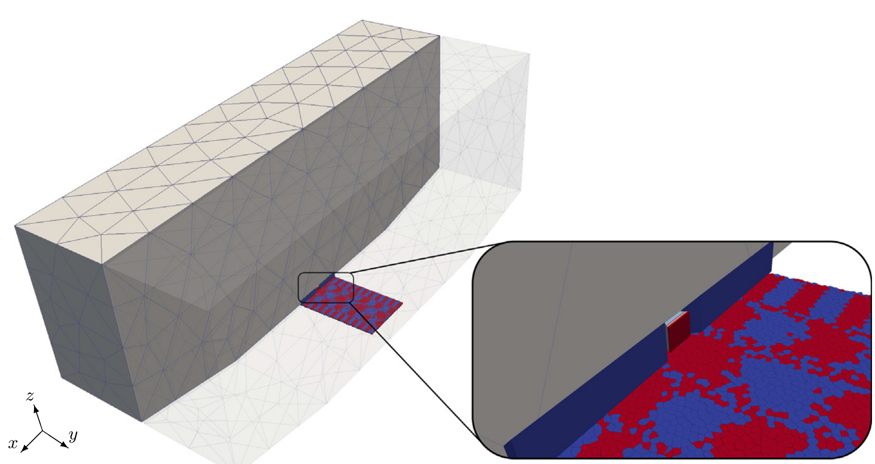

Fig. 2. The example setup consists of a GMR sensor element in between two macroscopic shields (5 μm × 2 μm × 2 μm). Beyond the GMR sensor a magnetic storage medium is indicated (it will not be considered for the calculation of the transfer curves).