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An approximate solution to a moving boundary problem with space–time fractional derivative in fluvio-deltaic sedimentation process

Rajeev *, **Mohan Singh Kushwaha, Ajay Kumar**

Department of Applied Mathematics, Indian Institute of Technology (Banaras Hindu University), Varanasi-221 005, India

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Abstract A mathematical model of the movement of the shoreline in a sedimentary ocean basin is discussed. The model includes space–time fractional derivative in Caputo sense and variable latent heat term. An approximate solution of the problem is obtained by Adomian decomposition method and the results thus obtained are compared graphically with an exact solution of integer order ($\beta = 1$, $\alpha = 1$). Three particular cases, the standard diffusion, the time-fractional and the space-fractional diffusions are also discussed. The model and solution are generalization of previous works.

1. Introduction

An interesting moving boundary problem in the field of earth surface science involves the movement of the shoreline in a sedimentary ocean basin (a shoreline problem). The classical diffusion transport models [1–3] provide a reliable means of modeling the sediment transport in fluvial depositional systems. The assumptions of the classical diffusion equation are thin-tailed periods of inactivity and thin-tailed transport distances for sediment particles. From the literature [4–7], the deviation from normal (Fickian) diffusion in sediment tracer dispersion is observed that violates the assumption of statistical convergence to a Gaussian. Therefore, the fractional diffusion equations are widely used for the investigation of the mechanism of anomalous diffusion in transport processes through complex and/or disordered systems including fractal media [8,9]. It is well known that fractional derivative is a good tool for taking into account memory mechanism, particularly in some diffusive processes [10]. Both space and time fractional operators correspond to the diffusion limit of continuous time random walk models with long-tailed waiting time and/or jump length distributions [10,11]. Li et al. [12] used Caputo derivative $\beta \in (0, 1]$ and Riesz-Feller derivative $\alpha \in (0, 2]$ operators for the first order time derivative and second order space derivative, respectively and presented an analytic solution to fractional form of a moving boundary problem in drug release devices in term of Fox H function. Voller [13] presented

* Corresponding author. Mobile: +91 9451941396.
E-mail addresses: rajeev.apm@itbhu.ac.in, rajeevbhu.mac@gmail.com (Rajeev).

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Nomenclature

h	height of the earth's crust (basement) above datum, m
x	space variable, m
t	time
$z(t)$	ocean level above datum, m
$s(t)$	shoreline position, m
$u(t)$	position of intersection between offshore sediment wedge and basement, m
q	prescribed sediment line flux, $m^3\ m^{-1}\ t^{-1}$
a	slope of off-shore sediment wedge
b	slope of basement

Greek symbols

η	height of sediment above datum, m
γ	a constant
ν	diffusion coefficient, $m^2\ t^{-1}$
α	fractional order of space derivative
β	fractional order of time derivative

fractional (non-integer) form of a limit Stefan problem using Caputo derivatives for both space and time, and discussed exact solution of the problem. Recently, some researchers [14–17] also discussed various mathematical models governed with different fractional derivatives for both the space and time.

The most commonly used definitions in mathematical models are the Riemann–Liouville and Caputo. Riemann–Liouville fractional derivative requires initial conditions to be expressed in terms of fractional integrals and their derivatives which have no obvious physical interpretation. So, Riemann–Liouville fractional derivative is not always the most convenient definition for real applications [18]. However, Caputo fractional derivative requires the initial conditions (including the mixed boundary conditions) in the same form as that of ordinary differential equations with integer derivatives [18]. These integer-order derivatives represent well-understood features of a physical situation and therefore their values can be measured accurately. Another advantage is that the Caputo derivative of a constant is zero, whereas the Riemann–Liouville fractional derivative of a constant is not zero. Therefore, it is interesting and applicable to use Caputo fractional derivative in diffusion model of sediment transport on earth surface. It can be seen in [19,20] that a pure power-law, heavy-tailed probability density function for the periods of inactivity without any truncation leads to a time-fractional diffusion equation which describes the evolution of surface elevation in time. Voller and Paola [21] presented the deviation of fluvial profiles from ones predicted by classical diffusion and proposed the exploration of fractional diffusive model to describe the observed steady-state fluvial profiles in a depositional system. Ganti et al. [22] discussed time fractional diffusion model for the surface dynamics of depositional systems by considering the fact that the periods of inactivity are heavy-tailed. They also discussed physical mechanisms constrain the occurrence of extremes in depositional systems and how these constraints reflect in the probability distributions of the random variables. They also presented that preliminary thoughts on continuum models for surface evolution of depositional systems are consistent with the documented probability distributions for erosional, depositional and inactivity events. Martin et al. [23] also discussed the physical basis for anomalous diffusion in bed load transport. Rajeev and Kushwaha [24] also discussed a mathematical model with time-fractional derivative for a moving boundary problem which occurs in sedimentation process. These models motivate to discuss space–time fractional diffusion model in sedimentation process to study the physical effect in complex domain.

The diffusion equation with a moving boundary (moving boundary problem) is a special nonlinear problem which is difficult to get the exact solution [25,26]. Hence, many approximate and numerical methods have been used to solve the moving boundary problems [27–33]. The approximate analytical approach taken in this literature is Adomian decomposition method (ADM). Adomian decomposition method was developed by Adomian [34–36] and has been applied to solve a wide class of non-linear differential and partial differential equations [37,38]. Grzymkowski and Slota [39] presented the solution of One-phase inverse Stefan problem by Adomian decomposition method. Das and Rajeev [29] also used and Adomian decomposition method to solve time-fractional diffusion equation with a moving boundary condition which is related to the diffusional release of a solute from a polymer matrix in which the initial loading is higher/lower than the solubility.

In this study, we consider the non-classical or non-Fickian, anomalous sediment transport in braided networks. Our attention in this paper is to discuss a moving boundary problem governed by fractional space–time derivative in Caputo sense which arises during the movement of the shoreline in a sedimentary ocean basin. The main physical purpose for adopting and investigating diffusion equations with fractional space–time derivative is to describe phenomena of anomalous (non-Fickian) sediment transport through complex and/or disordered systems including fractal media which occurs in sedimentation process. Adomian decomposition method is successfully applied to solve the proposed problem. The obtained results are compared with the existing exact solutions.

2. The fluvio-deltaic sedimentation model

Fluvio-deltaic sedimentation problem involves the shoreline propagation in a sedimentary ocean basin due to a sediment line flux, tectonic subsidence of the earth's crust, and sea level change. The mathematical model of fluvio-deltaic sedimentation process is discussed in [1–3]. In this paper, we consider a fixed line flux, a constant ocean level ($z = 0$), no tectonic subsidence of the earth's crust, and a constant sloping basement $b < a$. This scenario is a reasonable approximation for some modern continental margins. A schematic cross section of such a basin indicating the variables is revealed in Fig. 1 [2]. Under this limit case, the dynamics of the sedimentation process become a moving boundary problem with variable latent heat (see in [2]) which is as follows