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# Analytic description of domain-wall deformation caused by the Oersted field in current-driven domain-wall motion

Sang-Cheol Yoo, Kyoung-Woong Moon, Sug-Bong Choe *

CSO and Department of Physics, Seoul National University, Seoul 151-742, Republic of Korea

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ABSTRACT

We report here an analytic prediction of the domain-wall (DW) tilting caused by the Oersted field in the current-driven DW motion along ferromagnetic nanowires that have perpendicular magnetic anisotropy. By adopting the variational principle for energy minimization, the DW tilting angle is determined as a function of the current density with a finite threshold current density, above which the DW becomes elongated along the nanowire with two narrow domains at its edges. These results predict the minimum data bit size as well as the maximum current density needed for realizing stable DWs in DW-mediated nanodevices.



## 1. Introduction

Current-induced domain-wall (DW) motion in ferromagnetic nanowires [1,2] with perpendicular magnetic anisotropy (PMA) provides useful features [3,4] for DW-based devices such as magnetic random access memory and racetrack memory [5]. In these devices, the data bits are stored in the form of the DWs and the electric current is used to move the DWs to control their position. For a fast device operation, it is essential to use a high current density with these devices in order to induce a high DW speed [6]. However, the use of such a high current density induces a considerably strong Oersted field inside the devices, which in turn deforms the DWs. Such deformation limits the minimum size of the domains as well as the maximum current density required for achieving stable DW motion. In this study, we present an analytic theory on the effects of the Oersted field on the DW deformation.

This paper is organized as follows. In Section 2, we consider the simplest case of straight DWs. Then, in Section 3, the discussion is extended to arbitrarily shaped DWs for energy minimization. The numerical results are given in Section 4 to compare with the predictions of analytical calculations. Section 5 concludes this paper.

## 2. Straight DWs as the simplest case

A nanowire structure with a thin ferromagnetic layer sandwiched by two nonmagnetic layers is considered for this study, and its cross-sectional view is shown in Fig. 1(a). The width and thickness of the nanowire are denoted by $w$ and $t$, respectively. The ferromagnetic layer of thickness $t_m$ is centered at $z=0$. As shown in Fig. 1(b), a DW is initially positioned at $y=0$ in the nanowire. When a current is injected into the nanowire, the current generates a nonuniform Oersted field, resulting in the distortion of the DW. As the simplest case of such a distortion, we first consider a straight DW tilted by an angle $\theta$, as shown in Fig. 1(c). Generalization to a curved DW profile, as exemplified by Fig. 1(d), is discussed in Section 3. In this calculation, we use the coordinate convention that prescribes that the origin (i.e., $x=y=0$) is placed at the center of the DW, irrespective of the DW motion due to the translational symmetry along the nanowire. The width of the DW is assumed to be zero for simplicity and the effect of the finite DW width is discussed later. We consider here only the case in which the domain above (below) the DW is magnetized along $+z$ ($-z$) with the saturation magnetization $M_S$. The complementary case can be easily extended from the present case.

For an infinitesimal variation $\delta\theta$ in $\theta$, the variation $\delta E_O$ in the Zeeman energy due to the Oersted field is given by

$$\delta E_O = \mu_0 \int_{-t_m/2}^{t_m/2} \int_{-w/2}^{w/2} \int_{x\ \tan\ \theta}^{x\ \tan(\theta+\delta\theta)} 2M_S H_{OZ}(x,y,z)\, dy\, dx\, dz, \tag{1}$$

where $\mu_0$ is the magnetic permeability in vacuum and $H_{OZ}$ is the $z$-component of the Oersted field. Because $t_m \ll w$ for a typical geometry of lithography-patterned PMA nanowires such as Pt/Co/Pt films [7], the variation in $H_{OZ}$ along the ferromagnetic layer's thickness is negligible and allows the approximation $H_{OZ}(x,y,z) \cong H_{OZ}(x,y,0)$ inside the integral with respect to $dz$. Then, Eq. (1) can be rewritten as

$$\delta E_O = 2\mu_0 M_S t_m \sec^2\theta\, \delta\theta \int_{-w/2}^{w/2} H_{OZ}(x, x\ \tan\ \theta, 0) x\, dx. \tag{2}$$

* Corresponding author. Tel.: +82 2 880 9254; fax: +82 2 884 3002.
E-mail address: sugbong@snu.ac.kr (S.-B. Choe).

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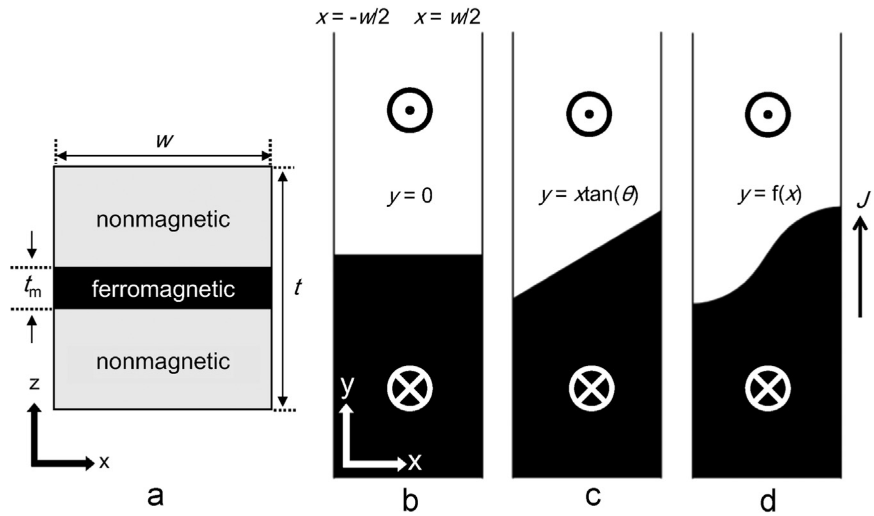


**Fig. 1.** (a) Cross-sectional view of the structure of the layer in these calculations. Planar views of (b) a straight DW at rest without current injection, (c) a straight DW with a tilting angle $\theta$, and (d) a curved DW with $y=f(x)$.

From the Biot–Savart law for steady currents along straight paths, $H_{OZ}(x,y,0)$ is given by

$$H_{OZ}(x,y,0) = -\frac{J}{2\pi}\int_{-t/2}^{t/2}\int_{-w/2}^{w/2}\frac{(x-x')}{(x-x')^2+(z')^2}dx'\,dz',$$

$$= \frac{J}{4\pi}\begin{cases} 2(w-2x)\tan^{-1}\left(\frac{t}{w-2x}\right)-2(w+2x)\tan^{-1}\left(\frac{t}{w+2x}\right) \\ +t\ln[t^2+(w-2x)^2]-t\ln[t^2+(w+2x)^2], \end{cases} \quad (3)$$

for the case of uniform current density $J$ across the nanowire's cross-section. After substituting Eq. (3) into Eq. (2) and integrating the latter, $\delta E_O$ can be written as

$$\delta E_O \cong -\frac{J}{2\pi}\mu_0 M_S t_m t w^2 \sec^2\theta\,\delta\theta, \quad (4)$$

up to the first leading term with respect to $t/w$.

The DW energy $E_W$ for a given $\theta$ is written as $E_W = \sigma_W t_m\sqrt{w^2+w^2\tan^2\theta} = \sigma_W w t_m \sec\theta$, where $\sigma_W$ is the DW energy density per unit area. The variation $\delta E_W$ in the DW energy is then easily obtained as

$$\delta E_W = \sigma_W w t_m \sec^2\theta\sin\theta\,\delta\theta, \quad (5)$$

for an infinitesimal variation $\delta\theta$. Note that the DW energy induces the DW tension, for which a shorter DW length is favorable.

For an infinitesimal variation $\delta\theta$ the variation $E_M$ in the magnetostatic energy due to the dipolar interaction is given by

$$\delta E_M = \frac{\mu_0}{2}\int_{-t_m/2}^{t_m/2}\int_{-w/2}^{w/2}\int_{x\tan\theta}^{x\tan(\theta+\delta\theta)} 2M_S H_{MZ}(x,y,z)\,dy\,dx\,dz, \quad (6)$$

where $H_{MZ}$ is the $z$-component of the dipolar field. The factor 1/2 comes from the nature of the self-interaction energy. As in the case of $H_{OZ}$, let us assume here that $H_{MZ}(x,y,z) \cong H_{MZ}(x,y,0)$ inside the integral. Then, Eq. (6) can be rewritten as

$$\delta E_M = \mu_0 M_S t_m \sec^2\theta\,\delta\theta\int_{-w/2}^{w/2} H_{MZ}(x, x\tan\theta, 0)x\,dx. \quad (7)$$

here $H_{MZ}(x,y,z)$ can be calculated by integrating the surface density of magnetic pole strength over the upper and lower interfaces

$$H_{MZ}(x,y,z) = \frac{M_S}{4\pi}\begin{cases} \int_{-w/2}^{w/2}\int_{-\infty}^{x'\tan\theta}[G^+(x,y,z;x',y')-G^-(x,y,z;x',y')]\,dy'\,dx', \\ -\int_{-w/2}^{w/2}\int_{x'\tan\theta}^{+\infty}[G^+(x,y,z;x',y')-G^-(x,y,z;x',y')]\,dy'\,dx', \end{cases} \quad (8)$$

where $G^{\pm}(x,y,z;x',y') = (z\pm t_m/2)/[(x-x')^2+(y-y')^2+(z\pm t_m/2)^2]^{3/2}$. This integration leads to

$$H_{MZ}(x, x\tan\theta, 0) = \frac{M_S}{\pi}[F^+(x)-F^-(x)], \quad (9)$$

where $F^{\pm}(x) = \tan^{-1}\left[\sqrt{t_m^2+(w\mp 2x)^2\sec^2\theta}/t_m\tan\theta\right]$. After substituting Eq. (9) into Eq. (7) and integrating the latter, $\delta E_M$ can be obtained as

$$\delta E_M \cong -\frac{\mu_0 M_S^2 w t_m^2}{2\pi}[\ln(4w/t_m)-1-f_d(\theta)]\sec^2\theta\sin\theta\,\delta\theta, \quad (10)$$

up to the first leading term with respect to $t_m/w$, where $f_d(\theta) = \ln(\cos\theta)-\ln(\sec\theta-\tan\theta)/\sin\theta$. Since the maximum variation in $f_d(\theta)$ is found to be about 0.3 (that is at least one order smaller than the typical values of $\ln(4w/t_m)-1$ in the calculations that follow, we replace $f_d(\theta)$ by the average value $\langle f_d(\theta)\rangle$ $(=\pi/2-\ln 2)$ over $[-\pi/2, \pi/2]$.

It should be noted that the dependence of $\delta E_M$ on $\theta$ is the same as the dependence of $\delta E_W$ on $\theta$. Thus, $\delta E_W+\delta E_M$ can be written in a unified form as

$$\delta E_W + \delta E_M = \sigma'_W w t_m \sec^2\theta\sin\theta\,\delta\theta, \quad (11)$$

by defining the effective wall energy density $\sigma'_W$ as

$$\sigma'_W \equiv \sigma_W - \frac{\mu_0 M_S^2 t_m}{2\pi}[\ln(4w/t_m)-1-\langle f_d(\theta)\rangle]. \quad (12)$$

For the case of 0.5-nm thick and 300-nm wide Pt/Co/Pt nanowires with typical magnetic parameters $\sigma_W = 4.0$ mJ/m$^2$ and $M_S = 1.4\times 10^6$ A/m, the effective wall energy density $\sigma'_W$ is estimated to be 2.8 mJ/m$^2$, which is 30% smaller as compared to the pure $\sigma_W$ due to the dipolar interaction. We note that a similar reduction is observed for the case of circular domains [8]. It is also worthwhile to confirm that the variation due to the replacement of $f_d(\theta)$ by $\langle f_d(\theta)\rangle$ is less than 0.1 mJ/m$^2$, which provides an upper bound on the error due to such replacement.