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Dual solutions for second-order slip flow and heat transfer on a vertical permeable shrinking sheet

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Abstract The aim of the paper is to study viscous fluid flow and heat transfer with second-order slip at linearly shrinking isothermal sheet in a quiescent medium. The sheet is permeable and subjected to constant suction. The governing equations consisting of the continuity, momentum, and the energy are transformed into a system of ordinary differential equations using suitable similarity transformation and solved numerically using the Runge–Kutta fourth-order method with the shooting technique. It is found that the problem possesses a dual physical solution. The effects of different parameters on the velocity and the temperature distributions as well as the skin-friction coefficient and the Nusselt number are presented graphically and in tabular form.

1. Introduction

A wide range of applications of nano-technology and micro-electro-mechanical systems have given a fillip to research area where a non-continuum behavior is present. In the present context, we are interested in studying surface-fluid interaction where slip flow regime occurs. In this regard, Kundsen number (K_n) is a deciding factor, which is a measure of molecular mean free path to characteristic length. When Kundsen number is very small, no slip is observed between the surface and the fluid and is in tune with the essence of continuum mechanics. However, when Kundsen number lies in the range 10^{-3} to 0.1, slip occurs at the surface-fluid interaction and is generally studied under the light of model Maxwell–Smoluchowski first-order slip boundary conditions. Adding, slip flow theory is an asset that enables to exploit Navier–Stokes equation even when the characteristic length approaches molecular mean free path. Slip flow theory has been validated by asymptotic solution of Boltzmann equation. In this analysis, inner kinetic solution is matched with outer (i.e., bulk) Navier–Stokes solution and the matching is obtained only when slip/jump coefficient are considered at boundary or at the surface (Hadjiconstantinou [1,2]). First and second slip flow coefficients are therefore outcomes of above said analysis. It is important to point that efficient slip flow model is always preferred, because of inherent simplicity over solution obtained for Boltzmann equation. The applicability of first-order slip model deteriorates as Kundsen is around or greater than 0.1. Therefore, a number

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of researchers have proposed second-order slip flow model. Wu [3] has proposed a second-order slip flow model for the flow of rarefied fluid along the surface based on numerical simulation of linearized Boltzmann equation. It is to mention in the light of above discussion that that either no slip regime or slip regime is taken as a boundary condition, the Navier–Stokes equation is still valid.

The flow induced by a moving surface has its importance in industrial applications and thus has been observed considerably in the literature. The permeable stretching/shrinking sheet is one such example, which has been studied with no slip regime or slip regime at the surface. Sakiadis [4], Tsou et al. [5], Crane [6], Chen and Char [7] contributed to the study of fluid flow along a moving/stretching surface. Magyari and Keller [8] presented an exact solution of self-similar boundary layer flow along a permeable stretching sheet. Andersson [9] obtained a solution of flow along stretching sheet with slip. Wang [10] analyzed the partial slip flow on stretching sheet in a quiescent medium. Miklavcic and Wang [11] studied flow on a shrinking sheet and argued the existence and non-uniqueness of a solution. Zhang et al. [12] obtained similarity solution of steady gaseous flow between parallel plates with first and second-order slip. Fang and Zhang [13] obtained an exact solution of MHD flow along a horizontal shrinking sheet without slip. Fang et al. [14] obtained an analytical solution of MHD flow along a shrinking sheet with first-order slip flow. Fang and Aziz [15] presented exact solutions of Navier–Stokes equation for second slip flow along a permeable stretching sheet. Yacob and Ishak [16] studied stagnation point flow of micropolar fluid over a shrinking sheet with a convective boundary condition. Rashidi et al. [17] studied flow of a second-grade fluid over a stretching or shrinking sheet using the multi-step differential transform method. Lok et al. [18] considered MHD flow along a shrinking sheet and inferred the existence of a dual solution for a small magnetic field. Fang et al. [19] presented an analytical solution for viscous flow along a shrinking sheet considering the second-order slip model presented by Wu [3]. Nanadeppanavar et al. [20] analyzed second-order slip flow over a horizontal shrinking sheet with a non-linear Navier boundary condition. Turkyilmazoglu [21] analytically studied heat and mass transfer in MHD viscous flow with hydrodynamic and thermal first-order slip over a stretching sheet for different thermal boundary condition. Turkyilmazoglu [22] presented analytically the existence of dual and triple solution in the flow of MHD viscoelastic fluid over a shrinking surface with first-order slip. Recently, Turkyilmazoglu [23] obtained analytically dual solution for MHD viscous flow with second-order hydrodynamic slip over a stretching/shrinking surface for non-isothermal/prescribed heat flux boundary condition.

In the present paper, we study viscous fluid flow and heat transfer with a second-order slip on a vertical isothermal shrinking sheet in a quiescent medium. The model of a second-order slip is the same as proposed by Wu [3].

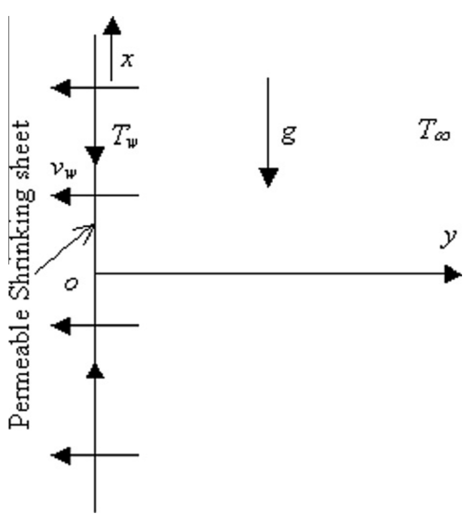

Figure 1 Problem schematic and coordinate system.

2. Formulation of the problem

Consider a continuous permeable vertical shrinking sheet at a temperature T_w with a linear velocity, $-cx$ ($c > 0$), and mass transfer velocity at the surface equal to v_w, in quiescent fluid of temperature T_∞. The x-axis is taken along the plate and the y-axis is taken perpendicular to the plate as is shown in Fig. 1. The fluid experiences a second-order slip at the sheet surface.

The governing equations of steady boundary layer flow are based on the continuity, momentum and the energy equations, which are given as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \pm g\beta(T - T_\infty), \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are the velocities along x and y directions, respectively, g is the acceleration due to gravity, β is coefficient of thermal expansion, ν is the kinematic viscosity, ρ is the fluid density, C_p is the specific heat at constant pressure, T is the fluid temperature, and κ is the thermal conductivity of fluid.

The boundary conditions for this problem are given by

Table 1 Comparison of $f''(0)$ with Fang et al. [19].

	Fang et al. [19]		Present paper	
	$S = 2.0$, $\lambda = 0.1$, $\delta = -1.0$	$S = 2.0$, $\lambda = 0.1$, $\delta = -2.0$	$S = 2.0$, $\lambda = 0.1$, $\delta = -1.0$, $Gr = 0.0$	$S = 2.0$, $\lambda = 0.1$, $\delta = -2.0$, $Gr = 0.0$
First Solution (F.S.)	0.34115	0.2037	0.3412	0.2038
Second Solution (S.S.)	0.3159	0.2655	0.3158	0.2656