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Deflection modeling of permanent magnet spherical chains in the presence of external magnetic fields

Kilian O'Donoghue, Pádraig Cantillon-Murphy *

School of Engineering, University College Cork, College Road, Cork, Ireland

A R T I C L E I N F O

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A B S T R A C T

This work examines the interaction of permanently magnetised spheres in the presence of external magnetic fields at the millimetre scale. Static chain formation and deflection models are described for N spheres in the presence of an external magnetic field. Analytical models are presented for the two sphere case by neglecting the effects of magnetocrystalline anisotropy while details of a numerical approach to solve a chain of N spheres are shown. The model is experimentally validated using chain deflections in 4.5 mm diameter spheres in groups of 2, 3 and 4 magnets in the presence of uniform magnetic fields, neglecting gravitational effects, with good agreement between the theoretical model and experimental results. This spherical chain structure could be used as an end effector for catheters as a deflection mechanism for magnetic guidance. The spherical point contacts result in large deflections for navigation around tight corners in endoluminal minimally invasive clinical applications.

1. Introduction

In this work, a model for the interaction of a chain of permanently magnetised spheres with external magnetic and gravitational potential fields is presented. The use of millimetre sized spheres as a magnetic navigation device in minimally invasive surgeries is explored. It has been found that chains of spherical magnets used as distal attachments can provide greater deflections for catheter devices than mechanical catheters in specific settings [1]. The greater deflections that are achieved with the current method allows the use of much lower strength steering magnets than those used in current magnetic navigation systems such as the commercially available Stereotaxis Niobe [1] and in research systems such as those presented by Martel et al. [2,3]. With a lower magnet strength requirement, a magnetic steering platform can potentially be made small enough to be a semi-portable system that can be easily setup in an operating room for a clinical procedure without modification to the room or impeding access to the patient with bulky equipment.

A classical physics formulation based on minimisation of the systems potential energy is employed. This is achieved analytically for simple cases (N=2) and numerically for larger numbers of spheres. In the static case, a system will approach a stable equilibrium position that minimises the potential energy of the system, or in other words, when all the forces and moments sum to zero [4].

A similar approach has been used by Stambaugh et al. to examine pattern formation in layers of permanent magnets [5,6] where numerous bar magnets were encapsulated in spheres and placed in layers. These layers were then shaken for a period until the layers reached an equilibrium position.

The majority of magnetic chain modeling to date has been on the micro- and nano-scales. For example, chain formation of ferromagnetic gold nanoparticles has been demonstrated experimentally [7] and self-assembly of magnetic nanoparticles on GaAs substrates have been shown to form chains at moderate applied fields of 0.7 T and form discrete clusters when exposed to large 17 T fields [8]. Dynamic modelling of the time response of magnetic particles has also been analysed when in nanowire [9] and particulate dispersion formations [10]. Similar methods have been used to analyse the electrostatic chain formation of lipid headgroups using Monte Carlo simulations [11].

The analysis of solid permanent magnets differs significantly from magnetic particles however. For example deformation of spherical droplets of magnetic particles into prolate ellipsoids significantly complicates the required calculations [12]. These effects can be neglected in solid permanent magnets. Also saturation and temperature effects are easily neglected for modern permanent magnet materials (e.g. NdFeB) under typical conditions [13]. Chains of permanently magnetised spheres have been used in the analysis of sound propagation due to the strong attraction force between the spheres [14] but there has been little or no research examining how a chain would interact with external

* Corresponding author. Tel.: +353 214903727.
E-mail addresses: kilianod@rennes.ucc.ie (K. O'Donoghue), padraig@alum.mit.edu (P. Cantillon-Murphy).

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fields. Beleggia et al. have presented an extensive set of papers exploring the force between permanent magnets in arbitrary shapes using potential energy formulations [15–17], but this was limited to force calculations and not shape formation. Magnetic catheter deflection has been demonstrated by Martel et al. where a catheter tip comprising of ferromagnetic spheres, which are free to rotate in compartments, has been steered using a modified MRI machine [2,18]. The Stereotaxis Niobe system makes use of cylindrical permanent magnets as a catheter tip and uses large steering permanent magnets to control the catheters position [1,19].

This paper is structured as follows. First, a theoretical model of the interaction between the spheres and external potential fields when constrained to a 2D plane is presented. Analytical expressions are derived for the two sphere case, while numerical simulations are described for chains of N spheres. These models are then tested experimentally with chains exposed to uniform magnetic fields up to 35 mT. Finally the results are discussed in the context of the clinical application to catheter steering.

2. Theoretical modeling

2.1. Spherical permanent magnets

Consider the magnetic field of a uniformly magnetised sphere of radius a. The magnetic field is given by Eq. (1) [20], where M_s is the volume magnetisation in A/m, r is the distance from the centre of the sphere in m, θ is the angle measured from the direction of magnetisation, $\hat{\mathbf{r}}$ is a unit vector pointing in the radial direction, $\hat{\boldsymbol{\theta}}$ is a unit vector in a clockwise sense about the x axis starting from $z=0$, and μ_0 is the magnetic permeability of free space:

$$\mathbf{B}(\mathbf{r}) = \begin{cases} \mu_0 \frac{2}{3} M_s & r<0 \\ \frac{\mu_0 M_s}{3}\frac{a^3}{r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}}) & r\geq 0 \end{cases} \quad (1)$$

The magnetic field outside the radius of the sphere is identical to the magnetic field resulting from an ideal magnetic dipole whose dipole moment is given by $m = \frac{4}{3}\pi a^3 M_s$. The dipole is aligned with the direction of the sphere's magnetisation. Hence in all calculations, the fields generated by each sphere are assumed to be an ideal dipole, which drastically simplifies the interaction equations.

2.2. Magnetic energy

If each magnet can be considered as a dipole, the standard Zeeman energy formula $U = -\mathbf{m}\cdot\mathbf{B}$ can be used to calculate the potential energy due to an external magnetic field. For the dipole–dipole interaction between each sphere, Eq. (2) may be used to determine the potential energy resulting from the interaction [21].

$$U_{dd}^{ij} = \frac{\mu_0}{4\pi r^3}[\boldsymbol{m_i}\cdot\boldsymbol{m_j} - 3(\boldsymbol{m_i}\cdot\hat{\mathbf{r}}_{\boldsymbol{ij}})(\boldsymbol{m_j}\cdot\hat{\mathbf{r}}_{\boldsymbol{ij}})] \quad (2)$$

In addition to the dipole interactions, the magnetocrystalline anisotropy of the material must also be considered. This anisotropy tends to move the magnetisation of a permanent magnet away from its easy axis to minimise its internal potential energy. The magnetocrystalline anisotropic energy of a magnet of volume V with a anisotropy constant K may be approximated by [20]

$$U_{ma} = VK\sin^2\gamma \quad (3)$$

where γ is the angle between the direction of magnetisation and the easy axis of the magnet. By summing the energy resulting from the external field, the dipole–dipole interaction and the magnetocrystalline anisotropy, the total energy of the system may be formulated, which in turn can be minimised to determine the final formation of a group of spheres. In this work, the anisotropy term will only be considered for the first sphere in the chain, which is assumed to be locked in place. Hence the only way for the sphere to minimise its potential energy is by shifting its magnetisation away from the easy axis. For all other spheres in the chain, it is assumed that their mechanical alignment minimises the potential energy and the anisotropy term given by Eq. (3) can be neglected.

2.3. Two sphere case

Consider a simple system with two spherical magnets in a 2D plane. The anisotropic energy term is initially neglected to facilitate an analytical solution for this simplified case. Fig. 1 shows two identical spheres of diameter D. The first sphere is rigidly fixed in place at the origin and aligned with the x-axis, while the second is free to rotate around the first magnet a diameter distance away. Its orientation is also free to rotate around its own principal axis. A uniform magnetic field is applied perpendicular to the alignment of the first sphere.

The position of the centre point of the second sphere is given by Eq. (4) and the magnetisation of each sphere is given by Eqs. (5) and (6)

$$\boldsymbol{r} = D(\cos\phi\hat{\boldsymbol{x}} + \sin\phi\hat{\boldsymbol{y}}) \quad (4)$$

$$\boldsymbol{m_1} = m_1\hat{\boldsymbol{x}} \quad (5)$$

$$\boldsymbol{m_2} = m_2(\cos\psi\hat{\boldsymbol{x}} + \sin\psi\hat{\boldsymbol{y}}) \quad (6)$$

Combining Eqs. (2), (4)–(6) the energy of the second sphere m_2 due to dipole–dipole interaction with the first sphere m_1 can be simplified to Eq. (7).

$$U^{int} = U_{dd}^{12} = \frac{\mu_0 m_1 m_2}{4\pi D^3}[\cos\psi - 3\cos\phi\cos(\phi-\psi)] \quad (7)$$

If the two spheres are exposed to a uniform magnetic field at right angles to the x-axis and the first sphere, m_1, remains fixed in space, the second sphere's potential energy is adjusted by the Zeeman contribution given in Eq. (8).

$$U_{ext} = -m_2 B\sin\psi \quad (8)$$

The Zeeman energy contribution of m_1 is neglected as it is fixed in position and its own potential energy does not affect the final position of the system. In order to determine the minimum energy position of the system, the partial derivative with respect to the two independent variables must be calculated, which results in Eqs. (9) and (10) when the magnetic strength of each sphere is

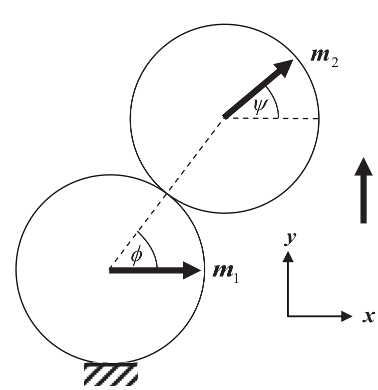

Fig. 1. In this analytical model for two magnetic spheres in contact, the first sphere is rigidly fixed in place and aligned with the x-axis. The centre point of the second sphere is free to move in a circular path to some angle ϕ relative to the first sphere and also to rotate around its own axis by some angle ψ. A uniform external magnetic field $\boldsymbol{B}$ is also present and points in the y direction.