Journal of Magnetism and Magnetic Materials 335 (2013) 36–45

Contents lists available at SciVerse ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Behavior of magnetic particles under fluctuating fields considering hysteresis characteristics

Naofumi Murata, Kenji Oguni *

Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama, Kanagawa 223-8522, Japan

A R T I C L E I N F O

Article history:
Received 9 July 2012
Received in revised form
27 December 2012
Available online 29 January 2013

Keywords:
Magnetic particle
Hysteresis
Field fluctuation
Clustering
Mechanical contact
Nonlinearity

A B S T R A C T

Simulations of magnetic particles are mostly carried out under spatially and temporally static magnetic fields. It is well known that chain-like clusters are formed under these constant fields. However the hysteresis characteristics of particles are therefore often neglected. In this paper, methods to analyze behavior of magnetic particles with hysteresis characteristics under fluctuating fields are proposed. Consideration of hysteresis characteristics and fluctuating fields brings nonlinearity and driving forces to the system in addition to the energy dissipation caused by particle collisions. This research aims to find the foothold of chaotic behavior, which is often found in these systems, among particle clustering. The proposed methods start from the discretization of fields. Collisions and clustering are modeled by treating mechanical contacts, namely, by solving Hertz's contact problem. A symplectic integrator is used for time integration. For the simulation, behavior of 200 magnetic particles under spatially and temporally fluctuating fields were examined. The results imply hysteresis characteristics have a great effect on clustering patterns.

1. Introduction

Analysis of magnetic particles' behavior is commonly adopted in the simulations of magnetic fluid and printer toners. In most cases, however, the assumed applied magnetic fields are constant and non-temporal [1,2]. Hence the hysteresis characteristics of each magnetic particle are often neglected or approximated as constant or linear, not considering the whole hysteresis loop. As far as the simulation methods are concerned, analyses are often carried out by means of free-energy theory or Monte Carlo simulations [3,4]. In case of free-energy theory, proper prediction of cluster patterns should be made, and the estimation of entropy would affect the results. Ewald Summation technique [5] used for Monte Carlo simulations requires periodicity of potential among the simulation cells and thus the effect of mutual interactions would become ambiguous, making the result blurry and averaged. These methods give practical results for some particular conditions, but would not in case under fluctuating fields with hysteresis characteristics. In large system where mutual interactions of particles appear strongly, nonlinearity of hysteresis and driving force from the field fluctuation along with the energy dissipation by collisions might bring chaotic behavior and patterns to the particles.

In this paper, methods to analyze the behavior of magnetic particles with hysteresis characteristics under spatially and temporally fluctuating fields are proposed. The method includes discretization of domain and fields, modeling of hysteresis characteristics, process for mechanical collisions of particles, and the integration of motion equations concerning each particle in the domain. Concrete processes for these steps are introduced in each section.

2. Calculation of field

In this section, the ways to treat magnetic fields in the simulation domain are discussed.

2.1. Analysis domain

Arbitrary magnetic field $\boldsymbol{H}$ is applied to the simulation area. Applied field magnetizes each particle according to its hysteresis curve, and then the mutual interactions between particles come into existence. In this paper, simulation area is divided into rectangular meshes with grids, and the magnetic vectors are discretely placed onto each grid point (Fig. 1). The simulation domain considered here is 2-D, but is enough for the assumed model which is 3-D spherical, magnetic particles fixed on the $x-y$ domain at their center points.

* Corresponding author. Tel.: +81 45 566 1834; fax: +81 45 566 1720.
E-mail addresses: naovillage@a8.keio.jp (N. Murata), oguni@sd.keio.ac.jp (K. Oguni).

0304-8853/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jmmm.2013.01.029

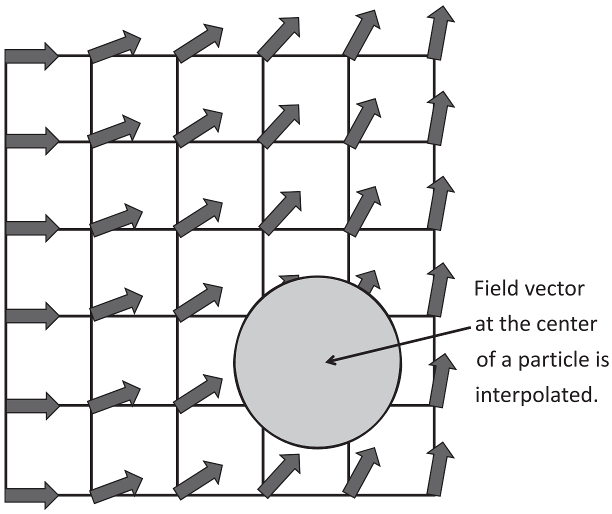

Fig. 1. Discrete field vectors assigned at the nodes of each mesh.

2.2. Interpolation between meshes

When a particle exists in the domain which is defined in Section 2.1, the potential energy U from the external magnetic field $\boldsymbol{H}$ is given by

$$U = -\boldsymbol{M}\cdot\boldsymbol{B} = -\mu_0 \boldsymbol{M}\cdot\boldsymbol{H}. \tag{1}$$

Here, $\boldsymbol{M}$ is the magnetic moment of a particle. Concrete ways to determine magnetic moments and magnetization processes are discussed in the next section. Particle's radius is set to 0.001 m and point-dipole approximation where the magnetic moment is assumed to be at the center of a particle is adopted. In this case, there is no need to consider demagnetization of each particle directly as long as the magnetization curve is corrected for demagnetization.

As Eq. (1) shows, potential energy of a particle U is given by $\boldsymbol{H}$, which is the field vector at particle's position in the meshed domain. As described above, field vectors are given discretely at each grid point, so that the interpolation by means of shape function is essential in order to determine the field vector of a particle placed in between the meshes. The shape function for rectangular-element-FEM [6] is adopted for this model.

In this simulation, calculation is simplified by parallel shift of the mesh; parallel shift by bringing each mesh's left below point to the origin. Size of the mesh is set to 0.001 m by 0.001 m and so that the shape function is written as follows:

$$u = u_1(1-100x-100y+10{,}000xy) + u_2(100x-10{,}000xy)+u_3(10{,}000xy) + u_4(100y-10{,}000xy). \tag{2}$$

As far as the field vector applied to a particle is concerned, when other particles exist in the domain, the total field vector of a particle strictly is a summation of pure external field $\boldsymbol{H}_f$ and $\boldsymbol{H}_{dipole}$. $\boldsymbol{H}_{dipole}$ here is the field created at its position from the other magnetized particles around. As a result, total field vector applied to a particle is given by

$$\boldsymbol{H} = \boldsymbol{H}_f + \sum^{N-1} \boldsymbol{H}_{dipole}. \tag{3}$$

Here N is the total number of particles in the domain.

In this paper, however, N is set to 200, which is so small that $\boldsymbol{H}_f$ becomes dominant and the effect of $\boldsymbol{H}_{dipole}$ can be assumed negligible; change in particles' magnetizations caused mutually by two dipoles in two-body system was calculated and the increase ratio was found out to be only 0.029% even under saturated field strength. Considering only $\boldsymbol{H}_f$ would reduce the total computation since $\boldsymbol{H}_{dipole}$ requires $o(n^2)$ iterative computation.

3. Magnetization and hysteresis of a particle

According to electromagnetism, an object in the magnetic field is magnetized. When the object is ferromagnetic, the magnetization characteristics become nonlinear and trace a hysteresis curve. In this section the modeling of hysteresis characteristics in the simulation is discussed.

3.1. Modeling of hysteresis curves

Hysteresis characteristics are often neglected in large scale finite element analysis of electromagnetic problems, due to its complexity and increase of computational costs. However in this simulation, mutual interactions between ferromagnetic particles under a fluctuating field are focused on, so that the consideration of hysteresis characteristics are of great importance. Several methods for modeling hysteresis such as Chua model and Preisach model are known, but here sigmoid function is chosen to express the hysteresis loop of γ-Fe_2O_3 [7] because of its simplicity.

Sigmoid function $\varsigma(x)$ is written as

$$\varsigma(x) = \frac{1}{1+e^{-ax}}. \tag{4}$$

With the appropriate parameters, the equations for upper and lower loops are, respectively, given as

$$\varsigma_{top}(x) = \frac{1.1}{1+e^{-6(x/10^5+0.15)}} - 0.55, \tag{5}$$

$$\varsigma_{bottom}(x) = \frac{1.1}{1+e^{-6(x/10^5-0.15)}} - 0.55 \tag{6}$$

to form the model of γ-Fe_2O_3 hysteresis loop. In this model, the saturation field strength is set to 1.2×10^5 A/m.

3.2. Magnetization process in the simulation

Simulating the exact magnetization process of a general polycrystal ferromagnetic substance considering domain wall motion and rotational magnetization is so complex and arcane that the ferromagnetic substance here is assumed to have the following characteristics:

- General hysteresis loop characteristic with domain wall motion and rotational magnetization.
- Spherical polycrystal and random orientation of crystals cancel the crystal anisotropy; a particle becomes isotropic as a total.
- Isotropic so that the magnetization turns to the direction of the applied field.
- Direction of magnetization remains even after the applied field is reversed until the norm of magnetization according to the hysteresis curve becomes 0; when the magnetization becomes below 0, the direction turns to the way of applied field.

Considering the total isotropy, hysteresis loop becomes origin symmetry. The hysteresis loop shown in Fig. 2 which is used in the simulation therefore is formed by evenly shifting the canonical sigmoid function to positive and negative directions on the x-axis. When actually determining the magnetization vector of a particle from this hysteresis model, first the norm of magnetization is calculated from the norm of field applied to the particle, and second the norm is multiplied by the unit vector of field direction. The second and third quadrants of hysteresis loop are