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Analytical solution of electrically conducted rotating flow of a second grade fluid over a shrinking surface

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Abstract The steady two-dimensional MHD rotating flow of a second grade past a porous shrinking surface is investigated. The governing system of partial differential equations is transformed into ordinary differential equations, which are then solved analytically by using the homotopy perturbation technique. The effects of the governing parameters on the flow field are obtained and discussed graphically in detail.

1. Introduction

The study of boundary layer flow due to a stretching sheet is important in industrial applications such as in extrusion processes. Sakiadis [1] did the pioneering work of boundary layer flow over a stretching sheet. Later, Tsou et al. [2] and Crane [3] studied the steady two-dimensional boundary layer flow over a stretching flat surface. Gupta and Gupta [4] extended this idea to include suction or blowing. On the other hand, boundary layer flow past a stretching sheet in a rotating fluid has been studied in various aspects by Wang [5], Rajeswari and Nath [6], Vajravelu and Kumar [7] and Nazar et al. [8] in Newtonian fluids, while in non-Newtonian fluids; Kumari et al. [9] analyzed such a flow of a power-law fluid. Besides that, Ali and Magyari [10] studied the unsteady two dimensional boundary layer flow and heat transfer stretching problem when the steady motion was slowed down gradually and Kumari and Nath [11] considered the unsteady flow at the axisymmetric stagnation point of a rotating body of revolution (sphere) with mass transfer, which is important in the study of spacecraft and missiles.

Recently, a paper by Miklavcic and Wang [12] investigated the flow over a shrinking sheet, where the velocity on the boundary is toward a fixed point, and found an exact solution of the Navier–Stokes equations. It was found that mass suction is required to maintain the flow over a shrinking sheet. From physical point of view, vorticity of the shrinking sheet is not confined within a boundary layer, and the flow is unlikely to exist unless adequate suction on the boundary is

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imposed [12]. This new type of shrinking sheet flow is essentially a backward flow as discussed by Goldstein [13]. It is worth mentioning to this end that important and new results on the flow induced by a shrinking sheet in a viscous fluid were recently presented by Fang [14], Wang [15], Fang and Zhang [16], Fang et al. [17,18]. There are two conditions that the flow toward the shrinking sheet is likely to exist, whether an adequate suction on the boundary is imposed (Miklavcic and Wang [12]) or a stagnation flow is considered (Wang [15]), so that the velocity of the shrinking sheet is confined in the boundary layer. As mentioned previously, solutions do not exist for larger shrinking impermeable sheet in an otherwise still fluid, since vorticity could not be confined in a boundary layer. However, with an added stagnation flow to contain the vorticity, similarity solutions may exist.

The field of MHD was initiated by Swedish physicist, Hannes Alfven for which he received in 1970 the Nobel Prize [19]. The official birth of incompressible fluid magneto-hydrodynamic is 1936–1937. In 1937, Hartmann and Lazarus [20] studied the influence of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid between two infinite parallel stationary and insulating plates. The most appropriate name for the phenomena would be Magneto-Fluid Mechanics, but the original name Magnetohydrodynamic is still generally used. MHD problems arise in a wide variety of situations ranging from the explanation of the origin of Earth's magnetic field and the prediction of space weather to the damping of turbulent fluctuations in semiconductor melts during crystal growth and even the measurement of the flow rates of beverages in the food industry. The description of MHD flows involves both the equations of fluid dynamics, the Navier–Stokes equations, and the equations of electrodynamics, Maxwell's equations, which are mutually coupled through the Lorentz force and Ohm's law for moving electrical conductors. Due to the coupling of the equations of fluid mechanics and electrodynamics, the equations governing MHD flows are rather cumbersome and exact solutions are, therefore, available only for some simple geometry subject to simple boundary conditions.

There are different methods in the literature to deal with such kinds of problems like the Adomian decomposition method [21], the variational iteration method [22–24], the Laplace decomposition method [25,26], the homotopy perturbation transform method [27] and the homotopy perturbation method (HPM) [28–30]. The homotopy perturbation method is the method, which is a coupling of the traditional perturbation method and homotopy in topology, deforms continuously to a simple problem which is easily solved. This method, which does not require a small parameter in an equation, has a significant advantage in that it provides analytical approximate solutions to a wide range of nonlinear problems arising in applied sciences [31–40]. Motivated by the above discussion, in this paper we consider MHD rotating flow over a shrinking surface. Present analysis has been carried out based on the homotopy perturbation method.

2. Formulation of the problem

Consider the MHD rotating flow of an incompressible second grade fluid over a shrinking surface at $z = 0$. Fluid occupying the space $z > 0$ is rotating with uniform angular velocity Ω, both fluid and plates are rotating with constant angular velocity Ω about the z-axis. The flow in the fluid system is caused due to shrinking of a plate. The following equations of continuity and momentum:

$$div = 0, \tag{2.1}$$

$$\rho\left[\frac{\partial V}{\partial t} + (V.\nabla)V + 2\Omega \times V + \Omega \times (\Omega \times r)\right] = divT + J \times B + R, \tag{2.2}$$

where $V = (u, v, w)$ is the velocity field, ρ is the density, R is the Darcy's resistance, J is the current density, B is the total magnetic field so that $B = B_0 + b$, b is the induced magnetic field (it is to be assumed that the induced magnetic field produced by the motion of an electrically conducting fluid is negligible compared to the applied magnetic field B_0) and r is a radial vector. The Cauchy stress tensor T for an incompressible second grade fluid is given by

$$T = -pI + \mu A_1 + \alpha_1 A_1 + \alpha_2 A_1^2, \tag{2.3}$$

in which T is the Cauchy stress tensor, p is the pressure, I is an identity tensor, μ is the dynamic viscosity, α_1, α_2 are the material constants and A_1, A_2 are the first two Rivilin Erickson tensors defined as

$$A_1 = L + L^T, \tag{2.4}$$

$$A_2 = \frac{dA_1}{dt} + A_1 L + L^T A_1, \tag{2.5}$$

$$L = \nabla V. \tag{2.6}$$

Furthermore, α_1 and α_2 satisfy the following constraints:

$$\mu \geqslant 0, \alpha_1 \geqslant 0, \quad \alpha_1 + \alpha_2 = 0. \tag{2.7}$$

Finally, continuity equation and momentum equation take the form:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{2.8}$$

$$u\frac{\partial u}{\partial x} + w\frac{\partial w}{\partial z} - 2\Omega = v\frac{\partial^2 u}{\partial z^2} + \frac{\alpha_1}{\rho}\begin{bmatrix} 2\frac{\partial^2 u}{\partial x \partial z}\frac{\partial u}{\partial z} + \frac{\partial^2 u}{\partial z^2}\frac{\partial w}{\partial z} + \frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2} \\ -\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial z^2} - \frac{\partial}{\partial z}\frac{\partial^2}{\partial x \partial z} - \frac{\partial}{\partial x}\frac{\partial^2}{\partial z^2} \\ +u\frac{\partial^2 u}{\partial x \partial z^2} + w\frac{\partial^3 u}{\partial z^3} \end{bmatrix} - \frac{\sigma B_0^2 u}{\rho} \tag{2.9}$$

$$u\frac{\partial}{\partial x} + w\frac{\partial}{\partial z} - 2\Omega = v\frac{\partial^2}{\partial z^2} + \frac{\alpha_1}{\rho}\begin{bmatrix} 2\frac{\partial w}{\partial z}\frac{\partial^2}{\partial z^2} + w\frac{\partial^3}{\partial z^3} + 2\frac{\partial w}{\partial z}\frac{\partial^2}{\partial z^2} \\ +u\frac{\partial^3}{\partial x \partial z^3} + 2w\frac{\partial^2}{\partial x \partial z}\frac{\partial u}{\partial z} \end{bmatrix} - \frac{\sigma B_0^2 u}{\rho} \tag{2.10}$$

with the boundary conditions

$$\begin{aligned} &u = u_w(x) = -cx, \quad v = 0, \quad w = -W, \quad \text{at } z = 0, \\ &u \to 0, \quad v \to 0 \quad \text{as } z \to \infty, \end{aligned} \tag{2.11}$$

in which $c > 0$ the shrinking constant and $W > 0$ the uniform suction velocity.

Introducing the following similarity transforms

$$u = cxf'(\eta), \quad v = cxg(\eta), \quad w = -\left(\frac{\mu c}{\rho}\right)^{\frac{1}{2}} f(\eta), \quad \eta = \left(\frac{\rho c}{\mu}\right)^{\frac{1}{2}} z, \tag{2.12}$$