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Magnetostatic Green's functions for the description of spin waves in finite rectangular magnetic dots and stripes

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ABSTRACT

We present derivation of the magnetostatic Green's functions used in calculations of spin-wave spectra of finite-size non-ellipsoidal (rectangular) magnetic elements. The elements (dots) are assumed to be single domain particles having uniform static magnetization. We consider the case of flat dots, when the in-plane dot size is much larger than the dot height (film thickness), and assume the uniform distribution of the variable magnetization along the dot height. The limiting cases of magnetic waveguides with rectangular cross-section and thin magnetic stripes are also considered. The developed method of tensorial Green's functions is used to solve the Maxwell equations in the magnetostatic limit, and to represent the Landau–Lifshitz equation of motion for the magnetization of a magnetic element in a closed integro-differential form.

1. Introduction

Recent advances in nano-fabrication technology resulted in the development of novel micro- and nano-structured materials with tuneable magnetic properties and submicron- and nano-scale components of magnetic devices. A quantitative understanding of the responses of nano-structured magnetic materials to driving microwave electromagnetic fields is necessary for the development of a new generation of frequency-agile microwave devices based on nano-structured artificial magnetic materials with properties that are not found in nature.

Nanomagnetic materials have a great potential for applications in modern technologies such as information storage, microwave and magnetic field sensing, biomedicine and spintronics. Reduction of the element size in these materials leads to the magnetization confinement and to the formation of new magnetic states, for example, magnetic vortices and localized spin wave excitations, which drastically change the fundamental dynamic properties of nano-structured materials. Therefore, magnetic dynamic processes at nano-scale, interesting by themselves, will be an important consideration in device design. Signal processing devices based on nano-structured magnetic materials and controlled by either magnetic field or spin-polarized current provide an opportunity to use spin waves (SW) (or magnons) as elementary information carriers. A research field in modern nanoscience that aims to use SW to store, carry and process information is called *magnonics* [1].

Usually, magnetic elements from which an artificial magnetic material is formed have a *non-ellipsoidal* shape. This means that even when the external magnetic fields acting on the elements are spatially uniform, the internal magnetic fields of the elements, determining the elements' static and dynamic magnetic properties, have a non-trivial coordinate dependence. These spatially non-uniform internal fields influence the spectra of magnetic excitations (spin waves) of the non-ellipsoidal magnetic elements and affect the dynamic magnetic properties of the elements. In particular, the dynamic magnetization in a non-ellipsoidal magnetic element can be strongly spatially localized, and the characteristic localization length could be substantially smaller that the geometric size of the element. At the same time, the finite geometric sizes of the magnetic element and the corresponding boundary conditions at the edges could lead to the quantization of SW excitation frequencies in the element. The boundary conditions at the element edges depend on the fundamental magnetic properties of the material such as exchange constant and volume and surface anisotropies, as well as on the element shape, edge geometry and roughness.

In most practically interesting cases the elementary magnetic dots are flat submicron-sized magnetic particles, in which the thickness is substantially smaller that the in-plane dot sizes. The dots usually have a shape of a circular [2,3] or an elliptical [4]

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cylinder, or a rectangular prism [5,6]. High-frequency dynamic properties of patterned magnetic films consisting of thin non-ellipsoidal magnetic elements have been investigated experimentally and theoretically during the last decade.

The experiments performed on uncoupled arrays of long axially magnetized magnetic stripes with rectangular cross-section have indicated that the excitation spectrum of transverse SW modes in such patterned film is quantized [5,6]. The geometrical quantization observed in Refs. [5,6] is a direct consequence of the boundary conditions at the lateral edges of the magnetic stripes forming a pattern. Similar results, i.e. the geometrical quantization of the SW spectrum, have been observed for arrays of non-interacting sub-micron-sized tangentially magnetized cylindrical and rectangular NiFe (Permalloy) dots [3,7]. The first observations, made by means of Brillouin light scattering spectroscopy (BLS), demonstrated existence of several discrete modes with the frequency splitting of up to 2.5 GHz [3]. These modes were identified as magnetostatic spin waves laterally quantized due to the finite in-plane sizes of each individual magnetic dot. Quantized standing spin waves of individual magnetic dots have been observed in micron sized thin-film elements not only using BLS, but also by means of the time-resolved Kerr effect and broadband ferromagnetic resonance [8–13]. The properties of arrays of non-interacting magnetic dot are intensively studied because of the possible applications of these arrays as patterned media for magnetic recording.

Currently the interest in the SW mode spectra of patterned magnetic films is renewed due to the possible applications of these films in the field of magnonics [1]. In the case of a relatively dense array of magnetic elements (or dots), the interdot dipolar coupling is expected to play a crucial role leading to formation of collective SW modes of an array and to the appearance of allowed (pass-) and forbidden (stop) frequency bands for the SW propagating in a coupled dot array.

It is known that properties of spin waves propagating in artificial magnetic periodic structures (or the so-called magnonic crystals) can be controlled by periodic modulation of structure parameters: thickness of a continuous magnetic film or width of a quasi-one-dimensional SW waveguide [8,9]. The variation of the structure parameters allows one to control the widths and frequency positions of allowed and prohibited bands in the SW spectrum. Periodic arrays of magnetic dots coupled by magnetostatic interaction can function as magnonic crystals, and recent studies of the static and dynamic properties of magnetic dots and dot arrays revealed many interesting properties of these systems (see e.g. [14]), which can be employed to create artificial microwave materials with novel functionalities.

The propagation of SW in planar periodic structures based on ferrite films has been investigated using the space- and time-resolved BLS and inductive detection techniques [8]. Measurements performed in arrays of uniformly magnetized stripes demonstrated the existence of several collective modes, propagating through the structure [8,10,11]. Multilayer films composed of magnetic layers having different magnetizations and planar arrays of magnetic stripes can also be considered as one-dimensional magnonic crystals [15]. The critical issue here is to create forbidden (stop) bands for the propagating SW. Clearly, dynamical properties of such periodic structures can be tuned by the variation of the external magnetic field, as well as by the variation of the structure geometrical sizes. The experimental investigations of the SW propagation and reflection in two-dimensional magnonic crystals formed by magnetic dot arrays are currently in progress (see e.g. [16]), and the broadband ferromagnetic resonance technique has been used recently to investigate influence of the interdot coupling on the SW spectra [17].

The problem of theoretical calculation of the SW spectra in non-ellipsoidal magnetic dots is important because the understanding of these spectra is necessary for the description of the magnetization switching in magnetic dots. The general theory of the SW spectra in flat magnetic elements based on the theory of the SW spectra in continuous magnetic films [18] was developed in Refs. [19,20]. The theory of SW modes localized in the regions of spatially non-uniform internal magnetic field near the edges of a non-ellipsoidal magnetic element is presented in Refs. [21,22]. Approximate boundary conditions for the variable magnetization at the edges of non-ellipsoidal magnetic elements have been derived by Guslienko et al. [23] and Guslienko and Slavin [24] and, then, have been successfully used for the description of the discrete SW modes in magnetic stripes [23,25,26], rectangular dots [27,28] and flat magnetic dots of other shapes. The SW modes in micro- and nano-sized magnetic dots have a mixed dipole-exchange nature when the lateral dot sizes or/and the characteristic localization lengths are comparable to the exchange length of the dot magnetic material. The main technical problem in the theoretical description of the SW spectra in the non-ellipsoidal magnetic elements is the calculation of the spatially non-uniform dipolar (or magnetostatic) fields inside these elements. One of the first attempts to take these fields into account explicitly has been undertaken by Bryant et al. [29].

The goal of our current paper is to formulate a general approach to the calculation of non-uniform dipolar (or demagnetizing) magnetic fields in non-ellipsoidal magnetic elements based on the formalism of the tensorial magnetostatic Green's functions [18]. In the framework of this approach the system of equations consisting of the Landau–Lifshitz equation of motion for the dot magnetization and Maxwell equations in the magnetostatic limit is reformulated as a single closed-form integro-differential equation, where differential operator of the second order has the exchange nature and the kernel of the integral operator is the tensorial Green's function obtained from the solution of the magnetostatic Maxwell equations in a particular geometry of the studied magnetic element [7,18]. In the following we shall ignore the magnetocrystalline anisotropy of the dot material, since the practically used magnetic dots are usually made of Permalloy, which is magnetically isotropic. However, in general, the anisotropy could be easily taken into account in the tensorial Green's functions formalism (see e.g. [30]).

The method of the tensorial magnetostatic Green's functions has been successfully used to describe SW spectra in isolated magnetic stripes [23,25,26], rectangular [22,27,28] and cylindrical magnetic dots in a saturated state [31,32], and for the dots in the spatially non-uniform vortex ground state [33–36].

The paper is organized as follows. The basic equations and the general expressions for the tensorial magnetostatic Green's functions are analyzed in Section 2. The particular cases, corresponding to the different geometries of the magnetic elements, are considered in Section 3. In the same section the examples of calculated spin wave spectra of finite-size non-ellisoidal magnetic elements are given. Conclusions are presented in Section 4.

2. Basic equations

To describe magnetic dynamic excitations (spin wave modes) in finite-size flat magnetic elements we use the traditional Landau–Lifshitz equation of motion for the magnetization [37]:

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma [\mathbf{M} \times \mathbf{H}_{eff}], \tag{1}$$

where γ is the gyromagnetic ratio, the effective field $\mathbf{H}_{eff}$ is defined as a variational derivative of the total energy W of the magnetic element:

$$\mathbf{H}_{eff}(\mathbf{r},t) = -\frac{\delta W[\mathbf{M}(\mathbf{r},t)]}{\delta \mathbf{M}}. \tag{2}$$