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ORIGINAL ARTICLE

Second law analysis for hydromagnetic couple stress fluid flow through a porous channel

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Abstract In this work, the combined effects of magnetic field and ohmic heating on the entropy generation rate in the flow of couple stress fluid through a porous channel are investigated. The equations governing the fluid flow are formulated, non-dimensionalised and solved using a rapidly convergent semi-analytical Adomian decomposition method (ADM). The result of the computation shows a significant dependence of fluid's thermophysical parameters on Joule's dissipation as well as decline in the rate of change of fluid momentum due to the interplay between Lorentz and viscous forces. Moreover, the rate of entropy generation in the flow system drops as the magnitude of the magnetic field increases.

1. Introduction

In recent times, there has been a renewed interest in the thermodynamic analysis involving channel fluid flows. This is due to the usefulness of the study in several renewable energy applications. For instance, it could be used to predict the efficiency of many thermal systems exchanging heat between two heat reservoirs and other Carnot systems. Moreover, in the energy generation, excessive energy is wasted or dissipated in the form of heat. Hence there is need to minimise the wastage by improving the energy of the system. Based on this fact, few research works have been reported in the literature. For example, Adesanya and Makinde [1] reported the entropy generation in couple stress fluid flowing steadily through a porous channel with slip at the isothermal walls. Similarly, Adesanya and Makinde [2] studied the entropy generation rate in the couple stress fluid flowing through a porous channel with convective heating at the walls. Also in the class of couple stress fluid, Makinde and Eegunjobi [3] investigated the inherent irreversibility of heat in steady flow of a couple stress fluid through a vertical channel filled with porous materials. Other important work on the entropy generation in a moving fluid includes [4–7].

In all the studies above on the thermodynamics analysis, the effect of magnetic field, placed in the transverse direction to the flow channel has been neglected. In reality, magnetic field plays a vital role in many industrial and thermal engineering applications. For instance, it is useful in controlling extremely hot moving fluid like molten steel and many more.

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Nomenclature

β_1	dimensionless Navier slip parameter at the lower wall
β_2	dimensionless Navier slip parameter at the upper wall
η	coefficient of couple stress
γ_1	Navier slip coefficient at the lower plate
γ_2	Navier slip coefficient at the upper plate
κ	thermal conductivity
μ	dynamic viscosity
Ω	parameter that measures the temperature difference between the two heat reservoirs
ρ	fluid density
σ_e	electrical conductivity
θ	dimensionless fluid temperature
a^2	couple stress parameter
B_0	magnetic field strength
Br	Brinkman number
c_P	specific heat at constant pressure
E_G	volumetric rate of entropy
G	dimensionless pressure gradient
h	width of the channel
H^2	Hartmann number
N_1	entropy generation due to heat transfer
N_2	entropy generation due to entropy generation due to fluid friction and ohmic heating
N_s	entropy generation number
Pr	Prandtl number
s	suction/injection parameter
T	fluid temperature
T_0	temperature at the lower plate
T_f	final fluid temperature
u	dimensionless fluid velocity
u'	fluid velocity
v_0	uniform suction/injection velocity
x', y'	cartesian coordinates
x, y	dimensionless cartesian coordinates

Several researchers have investigated the entropy analysis in hydromagnetic fluid flow in recent times. For example, Das and Jana [8] presented the second law analysis for magnetohydrodynamic incompressible fluid flow through a porous channel by imposing Navier slip conditions at the walls. Adesanya and Falade [9] analysed the inherent irreversibility in the flow of hydrodynamic third grade fluid through a channel saturated with porous materials. Similarly, the effect of hall current was presented in a study by Das and Jana [10]. Interested readers can see more interesting work on the influence of magnetic field on entropy generation rate in Refs. [11–20].

Motivated by studies in [8–20], the objective of the present study was to examine the influence of magnetic field and ohmic heating of the couple stress fluid on the entropy production within the flow channel, which has not been accounted for in the literature. The outcoming results are expected to enhance many industrial and thermal engineering processes whose working medium is a non-Newtonian fluid, with a view to minimise entropy generation which tends to deplete the amount of available energy for work.

To achieve this objective, flow governing equations are formulated, and non-dimensionalised and approximate solution of the dimensionless coupled non-linear boundary-value problem are obtained by using a semi-analytical Adomian decomposition method [21,22]. The choice of the method is due to the fact that the method does not require any linearisation, discretisation, use of initial guess or perturbation. These approximation solutions are used to compute the entropy generation rate and irreversibility ratio.

In the following section, the problem is formulated and non-dimensional analysis is also presented. Section 3 of the work contains the method of solution, results are presented and discussed in Section 4, while Section 5 concludes the paper.

2. Mathematical formulation

A fully developed hydromagnetic non-Newtonian fluid flow between two parallel plates is considered. The parallel plates are infinite, permeable and stationary relative to the fluid motion as shown in Fig. 1. We choose a 2-dimensional cartesian coordinate system with x-axis along the flow direction and y-axis orthogonal to the planes of the parallel plates, separated by width $y = h$. Fluid injection occurs at the lower plate at a uniform rate v_0, matched with a corresponding fluid suction at the upper plate. A constant magnetic field of strength B_0 is applied perpendicular to the direction of fluid flow. For most industrial applications, a non-chaotic fluid flow is desired such that the magnetic Reynolds number is very small, and since no external voltage is applied to the fully developed flow system, the induced magnetic field and Hall effect are negligible. We further assumed that all the fluid properties are constant and the Stokes constitutive model for the couple stresses is used. Consequently, the momentum and energy balance equations, with the local volumetric entropy generation rate (E_G) for the fluid flow can be written as follows [1,10]:

$$v_0\rho\frac{du'}{dy'} = -\frac{dP}{dx'} + \mu\frac{d^2u'}{dy'^2} - \eta\frac{d^4u'}{dy'^4} - \sigma_e B_0^2 u', \tag{1}$$

$$\rho c_P v_0\frac{dT}{dy'} = \kappa\frac{d^2T}{dy'^2} + \mu\left(\frac{du'}{dy'}\right)^2 + \eta\left(\frac{d^2u'}{dy'^2}\right)^2 + \sigma_e B_0^2 u'^2, \tag{2}$$

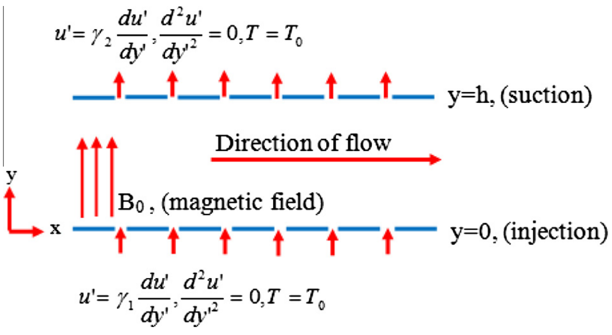

Figure 1 The geometry of the model.