Alexandria Engineering Journal (2016) 55, 951–958

HOSTED BY
ELSEVIER

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

Steady fully developed natural convection flow in a vertical annular microchannel having temperature dependent viscosity: An exact solution

Basant K. Jha, Babatunde Aina *, Zubairu Rilwanu

Department of Mathematics, Ahmadu Bello University, Zaria, Nigeria

Received 9 October 2015; revised 21 February 2016; accepted 25 February 2016
Available online 9 March 2016

KEYWORDS

Annular micro-channel;
Natural convection;
Temperature dependent viscosity;
Velocity slip and temperature jump

Abstract This study is devoted to investigate the steady fully developed natural convection flow in a vertical annular micro-channel having temperature dependent viscosity in the presence of velocity slip and temperature jump at the annular micro-channel surfaces. The governing equations of the motion are a set of ordinary differential equations and their analytical solutions in dimensionless form have been obtained for the temperature field and velocity field. The effect of various flow parameters entering into the problem is discussed with the aid of line graphs. During the course of numerical investigation, it is found that increase in viscosity variation parameter enhances the fluid velocity and velocity slip. Furthermore, an increase in viscosity variation parameter leads to increase in the volume flow rate and skin friction.

1. Introduction

Fluid flow in micro-channel has continued to attract interest because of its practical applications in space systems, manufacturing and material processing operations, and in high-power-density chips in supercomputers and other electronics. Several investigations have been accomplished on forced convection fluid flow in micro-channels and micro-tube. However, only a few studies have been carried out on natural and mixed convection in vertical micro-channels and micro-tube. Chen and Weng [1] studied the flow mechanism in a vertical micro-channel and obtained an exact solution of the fully developed natural convection in an open-ended vertical parallel-plate micro-channel due to asymmetric heating of micro-channel walls. They found that the rarefaction and fluid–wall interaction have significant effects on the flow and thermal fields. Jha et al. [2] extended this work by taking into account suction/injection on the micro-channel walls. They concluded in their work that skin friction as well as rate of heat transfer is strongly dependent on suction/injection parameter. The transient hydrodynamics and thermal behaviors of fluid flow in an open-ended vertical parallel-plate micro-channel, under the effect of the hyperbolic-heat-conduction model, were investigated semi-analytically in [3]. They concluded that, as Knudsen number increases, velocity slip and temperature jump increase at the boundaries. Haddad et al. [4] numerically investigated the developing hydrodynamical behaviors of free convection gas flow in a vertical open-ended parallel-plate

* Corresponding author.
E-mail addresses: basant777@yahoo.co.uk (B.K. Jha), ainavicdydx@gmail.com (B. Aina), zubairurilwanu72@gmail.com (Z. Rilwanu).

Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2016.02.023
1110-0168 © 2016 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V.

Nomenclature

B	viscosity variation parameter
C_{p_0}	specific heat at constant pressure
ln	fluid–wall interaction parameter, β_t/β_v
g	gravitational acceleration
k_1	radius of the inner cylinder
k_2	radius of the outer cylinder
Kn	Knudsen number, λ/w
q	volume flow rate
Q	dimensionless volume flow rate
Pr	Prandtl number
r	dimensional radial coordinate
R	dimensionless radial coordinate
$\widehat{R}$	specific gas constant
T	temperature of fluid
T_0	reference temperature
T_1	temperature at outer surface of the inner cylinder
u	axial velocity
U	dimensionless axial velocity
w	dimensional gap between the cylinders
σ_t, σ_v	thermal and tangential momentum accommodation coefficients, respectively

Greek letters

α	thermal diffusivity
β_0	coefficient of thermal expansion
β_t, β_v	dimensionless variables
γ	ratio of specific heats
μ	dynamic viscosity
μ_0	dynamic viscosity at $T = T_0$
θ	dimensionless temperature
ρ_0	density
v	fluid kinematic viscosity (μ_0/ρ_0)
η	ratio of radii (k_1/k_2)
λ	molecular mean free path
k_0	thermal conductivity
τ	skin-friction

micro-channel filled with porous media. Their results showed that the slip in velocity and jump in temperature decreased in the axial direction of the flow. Numerical solutions were obtained by Buonomo and Manca [5] for natural convection in parallel-plate vertical microchannels due to asymmetric heating by imposing constant heat flux on the boundaries. Buonomo and Manca [6] further performed a numerical study on transient mixed convection in a vertical micro-channel due to asymmetric as well as symmetric heat fluxes on the micro-channel surfaces. Chen and Weng [7] numerically studied the creep effect on the flow and heat transfer characteristics, for developing natural convective microflow in the same geometry.

In another related article, Weng and Chen [8] examined the influence of wall-surface curvature on the flow and thermal fields as well as the corresponding characteristics over the heated wall. The results of this work show the nonlinear behavior in temperature. It was also concluded that under certain rarefaction and wall interaction condition, by decreasing the curvature radius ratio, skin-friction decreases while rate of heat transfer increases. Jha et al. [9] extended the work of Weng and Chen [8] to the case in which the cylindrical surfaces forming the annulus are permeable, i.e. when there is a suction or injection through the annulus surfaces. They concluded that as suction/injection on the cylinder walls increases, the fluid velocity and temperature are enhanced. In another work, Jha and Aina [10] analyzed the work of Weng and Chen [8] by incorporating the pressure gradient in the vertical direction. It is observed that the probability of reverse flow formation increases at inner surface of outer cylinder of the micro-annulus by increasing curvature radius, while it decreases with increase in Knudsen number and fluid wall interaction parameter. Avci and Aydin [11,12] presented exact solutions for fully developed mixed convection in a vertical parallel-plate micro-channel with constant plate temperature and constant heat flux on the plates respectively. Also, Avci and Aydin [13] studied the fully developed mixed convective heat transfer of a Newtonian fluid in a vertical micro-annulus formed by two concentric micro-tubes. It is found that increasing mixed convection parameter enhances heat transfer while rarefaction effects considered by the velocity slip and the temperature jump in the slip flow regime decrease it. Jha and Aina [14] further extended the work of Avci and Aydin [13] to the case when suction/injection is imposed on the annulus surfaces. They concluded in their study that as suction/injection on the micro-porous-annulus (MPA) increases, the fluid velocity and temperature increase. Recently, Jha and Aina [15] investigated steady fully developed mixed convection flow in a vertical micro-annulus in the presence of transverse magnetic field. Das et al. [16] presented a theoretical analysis to investigate the effect of buoyancy force on mixed convective Couette flow of a reactive viscous incompressible nanofluid between two concentric cylindrical pipes under bimolecular, Arrhenius and sensitized reaction rates. In another article, Das et al. [17] studied the fully developed mixed convection flow in a vertical channel filled with nanofluids in the presence of a uniform transverse magnetic field. They reported that the magnetic field tends to enhance the nanofluid velocity in the channel. Makinde [18] investigated the thermal analysis of a reactive generalized Couette flow of power law fluids between concentric cylindrical pipes.

On the other hand, most of the existing analytical studies for such problems are based on the constant physical properties of the fluid. However, accurate prediction for the flow formation and heat transfer can be achieved by considering variation of physical properties with temperature [19], especially for fluid viscosity. Makinde and Chinyoka [20] presented numerical solution of unsteady flow of a variable viscosity reactive fluid in a slit with wall suction/injection. Tshehla et al. [21] investigated the entropy generation rate in a variable viscosity liquid flowing steadily through a cylindrical pipe with convective cooling at the pipe surface. They reported that a decrease in the fluid viscosity and an increase in viscous heating enhance total entropy generation in the flow fluid. Klemp