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ORIGINAL ARTICLE

Drag on a slip spherical particle moving in a couple stress fluid

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Abstract The creeping motion of a rigid slip sphere in an unbounded couple stress fluid is investigated. The linear slip boundary condition and the vanishing couple stress condition are applied on the surface of the sphere. A simple formula for the drag force acting on a slip sphere translating in an unbounded couple stress fluid is obtained. Special cases of the deduced drag formula are concluded and compared with analogous results in the literature. The normalized drag force experienced by the fluid on the slip sphere is represented graphically and the effects of slip parameter and viscosity coefficients are discussed.
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1. Introduction

The theory of couple stress fluids has been introduced by Stokes [1,2] to avoid the inadequacy of the classical Navier–Stokes model describing the correct behavior of some types of fluids with microstructure such as animal blood flow, chemical suspensions, and liquid crystals [3,4]. In the developed theory of fluids with couple stresses, it is considered that the surface of a portion of the fluid medium is affected on by a stress vector in addition to a couple stress vector [1]. The presence of non-symmetric stress tensor, body couples, and couple stress tensor, depending on the curvature twist rate tensor, distinguishes the model of couple stress fluids. Mathematically, the motion of a couple stress fluid is governed by an equation similar to the classical Navier–Stokes equation but with higher order and the constitutive equations are characterized by two tensors representing the stresses and the couple stresses [2].

Fluids with microstructure can also be described by another model, namely micropolar fluids, introduced by Eringen. Many authors have utilized the model of micropolar theory to physical problems of fluids with microstructure such as blood flow. A general expression for the drag force experienced by a micropolar fluid on an axisymmetric body has been derived by Ramkissoon and Majumdar [3]. They applied their derived formula to discuss the flow of micropolar fluid past a sphere. Hoffmann et al. [4] discussed the drag on a sphere in micropolar fluids assuming non-homogeneous boundary condition for the microrotation vector. Shu and Lee [5] derived fundamental solutions for micropolar fluids due to a point force and a point couple and applied their results to obtain the drag force acting on a solid sphere that translates in a low-Reynolds number micropolar flow. Deo and Shukla [6] studied the problem of creeping flow of a micropolar liquid past a fluid sphere with non-homogeneous boundary condition for microrotation. They discussed the drag force acting on the

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fluid sphere with respect to the material parameters. Ashmawy [7] obtained a general formula for the drag on a sphere placed in a creeping unsteady micropolar fluid flow.

Many other researchers have investigated viscous fluid flow problems through the use of Navier–Stokes equations. Felderhof [8] studied the steady motion of a viscous incompressible fluid with spinning particles using no-slip boundary condition. Lindgren [9] investigated the motion of a sphere in a viscous liquid at Reynolds numbers considerably less than one. Also, Liao [10] applied the homotopy analysis method to give approximate solutions for viscous fluid flow past a sphere using no-slip condition.

Although too many researchers considered viscous and micropolar fluid flows, less attention has been given recently to the model of couple stress fluids. Pralhad and Schultz [11] investigated the problem of blood flow in a stenosed tube using the couple stress model. Reddy et al. [12] studied the blood flow between the clogged artery and the catheter using the model of couple stress fluid. The same authors studied the problem of blood flow through unsymmetric stenosed tapered artery in the presence of catheter [13] using the model of couple stress fluids. Naduvinamani et al. [14] discussed the effect of surface roughness on the hydrodynamic lubrication of couple stress squeeze film between a sphere and a flat plate. Devakar and Iyengar [15] studied the motion of a couple stress fluid between two parallel plates. Iyengar and Vani discussed the slow rotational motion of a couple stress fluid between two confocal oblate spheroids [16]. The oscillatory flow of an incompressible couple stress fluid through an annulus with mild constriction at the outer wall is studied by Srinivasacharya and Srikanth [17]. Devakar et al. [18] applied the slip condition to the Couette and Poiseuille couple stress fluid flows.

The classical no-slip boundary condition has been applied extensively in the field of fluid dynamics. However, in the last century numerous studies have been established that the no-slip condition may not always occur and that the slippage of fluid particles on the surface of the rigid boundary can take place [19,20]. Recently, a linear slip boundary condition stating that the fluid tangential velocity relative to the solid boundary is proportional to the shearing stress acting at the contact point has been proposed and applied to many fluid problems either viscous or micropolar [21–29].

Based on the above mentioned literature review and to the best knowledge of the author, the influence of the drag force experienced by a couple stress fluid on a moving sphere, utilizing linear slip condition, has not been investigated yet. In the present work, we investigate the slow steady motion of a slip spherical particle in an unbounded incompressible couple stress fluid. The velocity slip boundary condition is applied on the surface of the rigid boundary. Also, it is assumed that the couple stress vanishes on the surface of the sphere. The drag force experienced by the couple stress fluid on the surface of the sphere using slip condition is obtained and represented graphically.

2. Formulation of the problem

The field equations governing a slow steady motion of an incompressible couple stress fluid, in the absence of body forces and body couples, are [1,2]

$$q_{i,i} = 0, \tag{2.1}$$

$$\mu q_{i,jj} - \eta q_{i,jjkk} - p_{,i} = 0, \tag{2.2}$$

where q_i is the velocity vector and p represents the fluid pressure at any point in the fluid flow. The classical viscosity coefficient μ has the dimensions M/LT while the couple stress viscosity parameters η and η' have the dimensions of momentum, namely ML/T. These material constants are constrained to the following restrictions [2].

$$\mu \geqslant 0, \eta \geqslant 0, \eta \geqslant \eta' \tag{2.3}$$

To completely describe the problem governed by (2.1) and (2.2), a set of six boundary conditions are needed. Three boundary conditions on the velocity, namely slip conditions, and three boundary conditions on couple stresses are provided.

The following slip boundary condition is imposed on the surface of the spherical particle, $r = r_0$, [22]

$$\beta_0 (q_i - U\hat{e}_z) = (\mathbf{I} - n_k n_k)(n_j t_{ji}) \quad on\ r = r_0, \tag{2.4}$$

where $\hat{e}_z$ is the unit vector along z-direction, n_j is the unit normal to the boundary pointing into the fluid and $\mathbf{I}$ is the unit dyadic. In addition, the parameter β_0 is termed the slip coefficient which is varying from zero to infinity. This parameter depends only on the nature of the fluid flow and the material of the boundary. The classical case of no-slip condition can be recovered as a special case when the slip parameter approaches infinity while the perfect slip is deduced when the slip parameter vanishes.

The remaining proposed boundary condition is the vanishing couple stress on the surface of the sphere. This means that the mechanical interaction at the boundary is equivalent to a force distribution only [2]

$$n_j m_{ji} = 0, \quad on\ r = r_0. \tag{2.5}$$

The stress tensor t_{ij} and the couple stress tensor m_{ij} can be written as [1,2]

$$t_{ij} = -p\,\delta_{ij} + 2\mu d_{ij} - \frac{1}{2} e_{ijk} m_{sk,s}, \tag{2.6}$$

$$m_{ij} = m\delta_{ij} + 4(\eta\,\omega_{j,i} + \eta'\,\omega_{i,j}), \tag{2.7}$$

where m is the trace of the couple stress tensor. The deformation rate tensor d_{ij} and the vorticity vector ω_i are given by

$$d_{ij} = \frac{1}{2}(q_{i,j} + q_{j,i}) \quad \omega_i = \frac{1}{2} e_{ijk}\, q_{k,j}. \tag{2.8}$$

Also, δ_{ij} and e_{ijk} are denoting the Kronecker delta and the alternating tensor, respectively.

Consider the steady creeping motion of an incompressible couple stress fluid extending to infinity; that is, the fluid region is assumed to be unbounded. We assume that a solid sphere of radius, r_0, is allowed to translate in an infinite region filled with a couple stress fluid. The velocity of the sphere is supposed to be constant of magnitude U directed along z-direction.

Working with the spherical polar coordinates (r, θ, ϕ), the velocity vector can take the form

$$\vec{q} = (u(r,\theta), v(r,\theta), 0). \tag{2.9}$$

The stream function $\psi(r, \theta)$ can be used such that

$$u(r,\theta) = \frac{-1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta}, \quad v(r,\theta) = \frac{1}{r\sin\theta}\frac{\partial \psi}{\partial r}. \tag{2.10}$$

Thus, the governing equations (2.2) reduce to

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