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# Numerical study of effect of induced magnetic field on transient natural convection over a vertical cone

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**KEYWORDS**

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Vertical cone

**Abstract** In the present paper, an analysis has been performed to study the influence of induced magnetic field on the transient free convective flow of an electrically conducting and viscous incompressible fluid over a vertical cone. The coupled nonlinear partial differential equations governing the transient flow have been solved numerically by using the implicit finite difference method of Crank–Nicolson type. The influence of magnetic parameter, magnetic Prandtl number and semi-vertical angle of the cone on the velocity and induced magnetic field profiles has been illustrated graphically. Also, the local as well as average skin-friction and Nusselt number has been presented graphically. For result validation, we have done a comparative study and the present results are found to be in very good agreement with available results.


## 1. Introduction

Free convective flow of an electrically conducting fluid in the presence of a magnetic field has been investigated substantially by many researchers because of its wide appearance in many industrial as well as technological applications. Heat transfer over a vertical cone in convective process is used for designing of engineering and scientific equipments such as nuclear waste disposal, nuclear reactor cooling systems, geothermal reservoirs [1]. In the present article, the system we are studying, a vertical cone with apex at the origin. The applied magnetic field is perpendicular to the surface along $y$ direction crossing the fluid motion. The resultant of this, the induced magnetic field is generated as shown in figure. Similarity solutions for axi-symmetrical problems for steady natural convective laminar flow over a vertical cone have been developed by many researchers [2–7]. Pop and Takhar [8] have analyzed the compressibility effects of the laminar free convective flow from a vertical cone while Watanabe [9] and Hossain and Paul [10] have studied the effect of different types of boundary conditions over a cone. Since, unsteady mixed convective flows do not specifically produce similarity solution; therefore, several flow problems have been analyzed, in which the non-similarity solutions were considered. There are a number of factors due to which unsteadiness and non-similarity occur in fluid flows such as velocities at edge of boundary layer, body curvature.

Anilkumar and Roy [11] have considered the unsteady mixed convection flow on a rotating cone in a rotating fluid due to the combined effects of thermal and mass diffusion. They obtained numerical results for the skin friction coefficients, Nusselt number and Sherwood number and presented

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**Nomenclature**

| | |
|---|---|
| $C_p$ | specific heat at constant pressure |
| $g$ | acceleration due to gravity |
| $H_0$ | constant applied magnetic field |
| $H'_1$ | induced magnetic field component along surface of cone |
| $H'_2$ | induced magnetic field component perpendicular to surface of cone |
| $H_1$ | dimensionless induced magnetic field component along surface of cone |
| $H_2$ | dimensionless induced magnetic field component perpendicular to surface of cone |
| $L$ | characteristic length |
| $M$ | magnetic parameter |
| $Nu$ | Nusselt number |
| $Nu_{av}$ | average Nusselt number |
| $Pm$ | magnetic Prandtl number |
| $Pr$ | Prandtl number |
| $r'$ | local radius of the cone |
| $T'$ | temperature of the fluid |
| $T$ | dimensionless temperature |
| $t'$ | time |
| $t$ | dimensionless time |
| $u'$ | velocity component along surface of cone |
| $v'$ | velocity component perpendicular to surface of cone |
| $u$ | dimensionless velocity component along surface of cone |
| $v$ | dimensionless velocity component perpendicular to surface of cone |
| $x'$ | distance along surface of cone |
| $y'$ | distance perpendicular to surface of cone |
| $x$ | dimensionless distance along surface of cone |
| $y$ | dimensionless distance perpendicular to surface of cone |

*Greek symbols*

| | |
|---|---|
| $\beta$ | coefficient of thermal expansion |
| $\mu_0$ | magnetic permeability |
| $\nu$ | kinematic viscosity |
| $\eta$ | magnetic diffusivity |
| $\phi$ | semi vertical angle of cone |
| $\rho$ | density |
| $\tau$ | local skin-friction |
| $\tau_{av}$ | average skin-friction |
| $\kappa$ | thermal conductivity of fluid |

the influence of various parameters on the velocity, temperature and concentration profiles. They also added that the self-similar solution can only exist, if the angular velocities at the edge and at the cone wall inversely vary as a linear function of time. Al-Harbi [12] performed the numerical analysis of natural convection heat transfer with effect of variable viscosity and thermal radiation from a cone and wedge in porous media and showed the effects of different pertinent parameters on the velocity, temperature and rate of heat transfer. Bapuji et al. [13] considered the finite difference analysis of laminar free convection flow past a non-isothermal vertical cone. They came to the conclusion that the difference between temporal maximum values and steady state values for both velocity and temperature becomes less when Prandtl number increases and the impact of Prandtl number on the local skin-friction and the local Nusselt number increases along the surface from the apex.

In all these studies, magnetic field has not been considered. It is found that magnetic field influences heat transfer processes of electrically conducting fluid flows. It has applications in industrial and technological fields such as metal coating, crystal growth, reactor cooling. Therefore, many researchers have shown their interest in studies of MHD free convective flow. MHD flow with heat flux is widely used in geothermal, geophysical and engineering fields. The geothermal gasses being electrically conducting are affected by magnetic field Bapuji and Chamkha [14] and Prakash et al. [15]. In many situations, variations in temperature become too large that variation in viscosity in thermal boundary layers cannot be neglected. This problem commonly appears in hot rolling process, production of glass fiber, plastic films drawing, drawing of wire and gluing of labels on hot bodies Kumar and Sivaraj [16]. In practical metallurgical processing where rates of extinguishing employ a significant effect on the finishing structure of products and unsteady hydromagnetic flows plays a big role. Takhar et al. [17] scrutinized the unsteady mixed convection flow from a rotating vertical cone with magnetic field. They concluded that tangential and azimuthal skin-friction coefficients and Nusselt number increase with time when angular velocity of cone increases.

Unsteady laminar heat and mass transfer from a rotating vertical cone with a magnetic field and heat generation or absorption effects were numerically investigated by Chamkha and Al-Mudhaf [18]. They found that the increase in buoyancy force increases the skin friction coefficients and the Nusselt number. Ece [19] studied free convection flow about a cone under mixed thermal boundary conditions and a magnetic field and he showed that the magnetic field retards the velocity profiles and expands the temperature profiles by increasing both the surface shear stress and temperature. The effect of different pertinent parameters on the velocity, temperature and rate of heat transfer was also discussed. Further, Ece [20] analyzed free convection flow about a vertical spinning cone under a magnetic field. He observed that the spin of the body expands the velocity profiles by increasing the skin friction and squeezes the temperature profiles toward the surface by increasing the surface heat flux for the specified surface temperature case and decreasing surface temperature for the specified heat flux case. The effect of mixed thermal boundary conditions and magnetic field on free convection flow about a cone in micropolar fluids was examined by Modather et al. [21] and they obtained transformation related to the similarity solutions of the boundary layer velocity, microrotation, and temperature profiles. Mahdy et al. [22] deliberated the study of the influence of magnetic field on natural convective flow near a wavy cone in porous media. It was found that magnetic field slows down