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ORIGINAL ARTICLE

Unaxisymmetric stagnation-point flow and heat transfer of a viscous fluid with variable viscosity on a cylinder in constant heat flux

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Non-uniform transpiration

Abstract Existing solutions of the problem of axisymmetric stagnation-point flow and heat transfer on either a cylinder or a flat plate are for incompressible fluid. Here, fluid with viscosity proportional to a linear function of temperature is considered in the problem of an unaxisymmetric stagnation-point flow and heat transfer of an infinite stationary cylinder with non-uniform normal transpiration $U_0(\varphi)$ and constant heat flux. The impinging free-stream is steady and with a constant strain rate $\bar{k}$. A reduction of Navier–Stokes and energy equations is obtained by use of appropriate similarity transformations. The semi-similar solution of the Navier–Stokes equations and energy equation has been obtained numerically using an implicit finite-difference scheme. All the solutions aforesaid are presented for Reynolds numbers, $Re = \bar{k}a^2/2\upsilon_\infty$, ranging from 0.01 to 100 for different values of Prandtl number and viscosity-variation parameter and for selected values of transpiration rate function, $S(\varphi) = U_0(\varphi)/\bar{k}a$, where a is cylinder radius and υ_∞ is the reference kinematic viscosity of the fluid. Dimensionless shear-stresses corresponding to all the cases increase with the increase in Reynolds number and transpiration rate function while dimensionless shear stresses decrease with the increase in viscosity-variation parameter. The local coefficient of heat transfer (Nusselt number) increases with increasing the transpiration rate function and Prandtl number.

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Nomenclature

a	cylinder radius
r	radial coordinate
z	axial coordinate
u, w	velocity components along (r, z)-axis
T	temperature
q_W	wall heat flux
T_∞	freestream temperature
$S(\varphi)$	transpiration rate function
k	thermal conductivity
$\bar{k}$	freestream strain rate
$f(\eta, \varphi)$	function related to u-component of velocity
Nu	Nusselt number
$U_0(\varphi)$	transpiration
Re	Reynolds number
Pr	Prandtl number
P	non-dimensional fluid pressure
p	fluid pressure
h	heat transfer coefficient
q_w	heat flow at wall

Greek symbols

η	similarity variable
φ	angular coordinate
α	thermal diffusivity
ρ	fluid density
υ	kinematic viscosity
υ_∞	reference kinematic viscosity
μ	viscosity of the fluid depending on the fluid temperature
μ_∞	viscosity of the ambient fluid
γ	viscosity-variation parameter
$\theta(\eta, \varphi)$	non-dimensional temperature
σ	shear stress

1. Introduction

The study of impinging jet problems has been of considerable interest during past decades because of great technical importance in many industrial applications, such as the drying of papers and films, the tempering of glass and metal during processing, the cooling of gas turbine surfaces and electronic components, surface painting, the water show technology and textile technology, pest-citing, de-icing, geology, biology, astrophysics and Chemistry. Existing solutions of the problem of axisymmetric stagnation-point flow and heat transfer on either a cylinder or a flat plate are for viscous, incompressible fluid. These studies were started by Hiemenz [1], who obtained an exact solution of the Navier–Stokes equations governing the two-dimensional stagnation-point flow on a flat plate, and were continued by Homann [2] with an analogous axisymmetric study and by Howarth [3] and Davey [4], whose results for stagnation-point flow against a flat plate for asymmetric cases were presented. Wang [5,6] was the first to find an exact solution for the problem of axisymmetric stagnation-point flow on an infinite stationary circular cylinder; this was continued by Gorla's works [7–11], which are a series of steady and unsteady flows and heat transfer over a circular cylinder in the vicinity of the stagnation-point for the cases of constant axial movement and the special case of axial harmonic motion of a non-rotating cylinder. Cunning et al. [12] have considered the stagnation-point flow problem on a rotating circular cylinder with constant angular velocity; Grosch and Salwen [13] as well as Takhar et al. [14] studied special cases of unsteady viscous flow on an infinite circular cylinder. The most works of the same types are the ones by Saleh and Rahimi [15] and Rahimi and Saleh [16,17], which are exact solution studies of a stagnation-point flow and heat transfer on a circular cylinder with time-dependent axial and rotational movements, as well as studies by Abbasi and Rahimi [18–21], which are exact solutions of stagnation-point flow and heat transfer but on a flat plate. Some existing compressible flow studies but in the stagnation region of bodies and by using boundary layer equations include the study by Subhashini and Nath [22] as well as Kumari and Nath [23,24], which are in the stagnation region of a body, and works of Katz [25] as well as Afzal and Ahmad [26], Libby [27], and Gersten et al. [28], which are all general studies in the stagnation region of a body. Recently, Alizadeh et al. [29–30] have considered the unaxisymmetric stagnation-point flow and heat transfer of a viscous fluid on a stationary and moving cylinder with time-dependent axial velocity and magnetohydrodynamic effects. Sheikholeslami et al. [31–39] have presented the thermal radiation and magnetohydrodynamics and entropy generation and space dependent magnetic field and ferrohydrodynamic of nanofluid and ferrofluid from a plate and between two horizontal parallel plates and in a semi annulus enclosure with viscous dissipation and free and force convection. Also, Kandelousi [40] have considered the effect of spatially variable magnetic field on ferrofluid flow and heat transfer considering constant heat flux boundary condition using finite element method. In a geological context, Ribe and Smooke [41] presented a two-dimensional dynamical model for melt extraction from a mantle plume and indicated that the flow in the melt zone has the form of a stagnation-point flow. Kellogg and Turcotte [42] also modeled the homogenization of the subducted oceanic crust with the depleted mantle considering the combined problem of thinning and diffusion at a stagnation-point flow. Studies of analytical investigation of MHD Jeffery–Hamel nanofluid flow in non-parallel walls were performed by Sheikholeslami et al. [43] who obtained solutions using a fourth order Runge–Kutta method.

All the aforesaid studies were confined to the fluid with constant viscosity. However, it is known that this physical property may change significantly with temperature. To predict accurately the flow behavior, it is necessary to take into account this variation of viscosity. On assuming that the viscosity of the fluid is linear functions of temperature, a semi-empirical formula was proposed by Charraudeau [44] which is appropriate for small Prandtl number. Studies of the effect of variable viscosity on flow and heat transfer to a continuous moving flat plate were performed by Pop et al. [45] and Pantokratoras [46] who obtained similarity solutions considering that viscosity varies as an inverse function of temperature.