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ORIGINAL ARTICLE

Non-linear thermal convection in a Casson fluid flow over a horizontal plate with convective boundary condition

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Suction

Abstract Casson fluid flow has many practical applications such as food processing, metallurgy, drilling operations and bio-engineering operations. In this paper, we study Casson fluid flow through a plate with a convective boundary condition at the surface and quantify the effects of suction/injection, velocity ratio, and Soret and Dufour effects. Firstly we used a similarity transformation to change the governing equations to ordinary differential equations which were then solved numerically. The effect of the rheological parameters on the velocity, temperature, and concentration with skin friction, and heat and mass transfer are shown graphically and discussed briefly. It is observed that the velocity of the fluid at the surface decreases with increase of the velocity ratio while the nature of the flow is in opposite characteristics. The local Nusselt number decreases with increase in the velocity ratio. Skin friction at the surface is enhanced by buoyancy ratio and Casson number. Due to injection of the fluid in the system, the mass transfer rate at the surface increases while it decreases with the velocity ratio parameter.

1. Introduction

Heat and mass transfer characteristics and convection in a non-Newtonian fluid have practical applications in many engineering and medical sciences [1]. Casson fluid is classified as a most popular non-Newtonian fluid due to its rheological characteristics [2–6]. Kameswaran et al. [7] have discussed the Casson fluid flow over a stretching or shrinking sheet. Mahanta and Shaw [8] have studied a three dimensional Casson fluid flow past a porous linearly stretching sheet in the presence of convective boundary condition. The influence of the viscous dissipation on the MHD Casson fluid flow over a non-Darcy porous medium has been discussed by Makanda et al. [9]. Casson fluid flow and heat transfer past an

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exponentially porous stretching surface have been discussed by Pramanik [10]. Convective boundary condition effect on the stagnation point flow of nanofluid has been discussed by Akbar et al. [11].

The processes of suction and injection find application in many industries such as in the design of thrust bearing and radial diffusers, and thermal oil recovery. Hartnett [12] analyzed the significance of suction or injection on boundary layer flow and noted that suction or injection through the surface can significantly modify the flow field. The effect of the suction or injection on the boundary layer flow and heat transfer has been studied by Pantokratoras [13]. The influence of the thermophoretic and non-linear convection has been studied by Kameswaran et al. [14]. The non-linear convection in nano-fluid flow on stretching sheet has been studied by Shaw et al. [15].

When heat and mass transfer occurs simultaneously in a moving fluid, the relation between the fluxes and the driving potential becomes complex [16–18]. The Soret effect (or thermal diffusion) is mainly the occurrence of a diffusion flux due to a temperature gradient, whereas the Dufour effect is due to a heat flux as a result of chemical potential gradients. Such effects are significant in the fields of geosciences and chemical engineering. Hayat et al. [19] studied the magnetohydrodynamics flow of a Casson fluid over a stretching surface with Soret and Dufour effects. Convective boundary conditions are applicable in industries with high temperatures such as in gas turbines, nuclear plants, and thermal energy storage. Non-Newtonian fluids are used in place of Newtonian fluids in these industries [20–27].

Heat transfer at the surface of the microvessels is followed by convective heat because of the connective tissue which covers the outer wall of the microvessel. Diffusion and heat flux play a vital role during atherosclerosis, hyperthermia and other diseases. The aim of the present problem is to study the influence of suction or injection due to the permeable surface, Soret and Dufour effects due to mass and heat flux on Casson fluid flow through a plate with a convective boundary condition at the surface. We transformed the governing equations to a system of ordinary differential equation using similarity variables. The transformed equations were solved numerically for different values of the velocity ratio, suction/injection parameter, and the Soret and Dufour parameters. This paper is very useful in medical science to know the flow nature of the blood in the vessel.

2. Mathematical formulation

We consider steady, incompressible Casson fluid flow over a heated flat plate. It is assumed that the free stream moves with a constant velocity U_∞ over the top of the surface and the plate moving opposite or same direction of the free stream with velocity U_w.

By assuming homogeneity and thermal equilibrium, the governing equations are written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + g\beta_c(C - C_\infty) + g[\beta_0(T - T_\infty) + \beta_1(T - T_\infty)^2], \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2} + \frac{\mu}{\rho C_p}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

The corresponding boundary conditions at the surface and far away from the surface are written as follows:

$$\begin{aligned} &u = U_w, v = V_w(x), -k\tfrac{\partial T}{\partial y} = h_f(T_f - T), C = C_w \quad && \text{at } y = 0, \\ &u \to U_\infty, T \to T_\infty, C \to C_\infty && \text{as } y \to \infty, \end{aligned} \tag{5}$$

where the velocities of the fluid u and v are along the x and y-axis, respectively, v is the apparent viscosity of the Casson fluid, g is the acceleration due to gravity, β_0 and β_1 are the linear and non-linear volumetric thermal expansion coefficient, β_c is the volumetric solute expansion coefficient, T is the temperature of the fluid, T_∞ is uniform ambient temperature, C is the concentration of the fluid, α_m is the thermal diffusivity, D_m is the effective solutal diffusivity of the medium, K_T is the thermal diffusion ratio, C_s is the concentration susceptibility, and C_p is the specific heat capacity. Convective boundary condition is introduced at the surface of the plate. It is assumed that the bottom surface of the plate is heated by convection from a hot fluid of temperature T_f which is related with the heat transfer coefficient h_f where k and T_w are the thermal conductivity and the uniform temperature at the wall, respectively. It is noted that $T_f > T_w > T_\infty$. Following the work of Ishak [28], we define the heat transfer coefficient as $h_f = cx^{-1/2}$ with constant c and corresponding Biot number written as $\mathrm{Bi} = \frac{c}{k}\sqrt{U/\alpha_m}$. The variable plate surface permeability function is given as

$$V_w(x) = -\frac{f_w}{2}\sqrt{\frac{U v_f}{x}} \tag{6}$$

where $U = U_w + U_\infty$. f_w is a constant with $f_w > 0$ representing the transpiration (suction) rate at the plate surface, $f_w < 0$ corresponds to injection and $f_w = 0$ for an impermeable surface. We introduce the following similarity transformation to convert the partial governing differential equation to system of ordinary differential equation, written as [29,30]

$$\eta = y\sqrt{\frac{U}{vx}}, \psi = \sqrt{vxU}f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \tag{7}$$

where ψ is the stream function which is defined as usual way $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which by default satisfied the continuity equation. So the remaining governing Eqs. (2)–(4) are written as

$$\left(1+\frac{1}{\beta}\right)f''' + \frac{1}{2}ff'' + \lambda(\theta + \alpha\theta^2 + N\phi) = 0, \tag{8}$$

$$\theta'' + D_f\mathrm{Pr}\phi'' + \frac{1}{2}\mathrm{Pr}f\theta' + \mathrm{Br}\left(1+\frac{1}{\beta}\right)f''^2 = 0, \tag{9}$$

$$\phi'' + \mathrm{SrSc}\theta'' + \frac{1}{2}\mathrm{Sc}\,f\,\phi' = 0, \tag{10}$$

and the corresponding boundary conditions are written as

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