Alexandria Engineering Journal (2016) 55, 1333–1339

HOSTED BY

ELSEVIER

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

Heat transfer analysis of boundary layer flow over hyperbolic stretching cylinder

Abid Majeed *, **Tariq Javed**, **Irfan Mustafa**

Department of Mathematics and Statistics, International Islamic University, Islamabad 44000, Pakistan

Received 23 October 2014; revised 15 March 2016; accepted 19 April 2016
Available online 9 May 2016

KEYWORDS

Boundary layer flow;
Hyperbolic stretching cylinder;
Heat transfer analysis;
Entropy generation

Abstract In the present study heat transfer and entropy generation analysis of boundary layer flow of an incompressible viscous fluid over hyperbolic stretching cylinder are taken into account. The governing nonlinear partial differential equations are normalized by using suitable transformations. The numerical results are obtained for the partial differential equations by a finite difference scheme known as Keller box method. The influence of emerging parameters namely curvature parameter and Prandtl number on velocity and temperature profiles, skin friction coefficient and the Nusselt number is presented through graphs. A comparison for the flat plate case is given and developed code is validated. It is seen that curvature parameter has dominant effect on the flow and heat transfer characteristics. The increment in the curvature of the hyperbolic stretching cylinder increases both the momentum and thermal boundary layers. Also skin friction coefficient at the surface of cylinder decreases but Nusselt number shows opposite results. Temperature distribution is decreasing by increasing Prandtl number. Moreover, the effects of different physical parameters on entropy generation number and Bejan number are shown graphically.

1. Introduction

The boundary layer flow and heat transfer with stretching boundaries received remarkable attention in modern industrial and engineering practice. The attributes of end product are greatly reliant on stretching and rate of heat transfer at final stage of processing. Due to this real-world importance, interest developed among scientists and engineers to comprehend this phenomenon. Common examples are the extrusion of metals into cooling liquids, food, plastic products, the reprocessing of material in the molten state under high temperature. During this phase of manufacturing process, the material passes into elongation (stretching) and a cooling process. Such types of processes are very handy in the production of plastic and metallic made apparatus, such as cutting hardware tools, electronic components in computers, rolling and annealing of copper wires. In many engineering and industrial applications, the cooling of a solid surface is a primary tool for minimizing the boundary layer. Due to these useful and realistic impacts, the problem of cooling of solid moving surfaces has turned out to be an area of concern for scientists and engineers. Recently researchers have shown deep interest in analyzing the characteristics of flow and heat transfer over stretching surfaces. In this context, analysis of flow and heat transfer phenomena over a stretching cylinder has its own importance in processes

* Corresponding author. Tel.: +92 3335251806.
E-mail address: abidmajeed726@yahoo.com (A. Majeed).
Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2016.04.028
1110-0168

such as fiber and wire drawing, hot rolling. Based on these applications, the boundary layer flow and heat transfer due to a stretching cylinder was initially studied by Wang [1]. He extended the work of Crane [2] to study the flow of heat transfer analysis of a viscous fluid due to stretching hollow cylinder. During last few years many researchers gave attention to analyze the fluid flow and heat transfer over the stretching cylinder and found the similarity solutions. Nazar et al. [3–5] produced numerical solution of laminar boundary layer flow, uniform suction blowing and MHD effects over stretching cylinder in an ambient fluid. After that a lot of research problems have been done on the analysis of flow and heat transfer over stretching cylinder. Mukhopadhyay [6–9] considered chemical solute transfer, mixed convection in porous medium, and MHD slip flow along a stretching cylinder. Effect of magnetic field over horizontal stretching cylinder in the presence of source/sink with suction/injection is studied by Elbashbeshy [10]. Fang and Yao [11], Vajravelu et al. [12], Munawar et al. [13,14], Wang [15], Lok et al. [16], Abbas et al. [17], and Butt and Ali [18] have considered viscous swirling flow, axisymmetric MHD flow, unsteady vacillating flow, natural convection flow, axisymmetric mixed connection stagnation point flow, radiation effects in porous medium and entropy analysis of magnetohydrodynamics flow over stretching cylinder, respectively. More recently, Majeed et al. [19] applied Chebyshev Spectral Newton Iterative Scheme (CSNIS) and studied the heat transfer influence by velocity slip and prescribed heat flux over a stretching cylinder. Sheikholeslami [20] has calculated the effective thermal conductivity and viscosity of the nanofluid by Koo–Kleinstreuer–Li (KKL) model and provided the nanofluid heat transfer analysis over cylinder. Javed et al. [21] discussed the combined effects of MHD and radiation on natural convection flow over horizontal cylinder. Hajmohammadi et al. [22] considered flow and heat transfer of Cu-water and Ag-water nanofluid over permeable surface with convective boundary conditions. Their main focus was to discuss the effects of nano-particles volume fraction, and non-dimensional permeability parameter on skin friction and convection heat transfer coefficient. Dhanai et al. [23] investigated mixed convection nanofluid flow over inclined cylinder. They utilized Buongiorno's model of nanofluid and found dual solutions with the velocity and thermal slip effects in the presence of MHD.

Aforementioned studies reveal that vast literature is present on study of linearly stretching cylinder but to the best of our knowledge, the problem over a cylinder with hyperbolic stretching velocity and entropy generation has not been addressed yet. The governing equations related to flow problem solved numerically and the effects of involving parameters on flow behavior are shown graphically. Entropy generation is the quantification of thermodynamics irreversibility and it exists in all types of heat transfer phenomenon. Due to this fundamental importance the present work is optimized with the inclusion of entropy generation analysis. In this context the studies [24–30] are quite useful to explore many aspects of entropy generation which have not been taken into account yet.

2. Problem formulation

Consider the two-dimensional steady incompressible flow of a viscous fluid over a hyperbolic stretching circular cylinder of fixed radius R (see Fig. 1). The basic equations that govern the flow and heat transfer phenomena will take the form as

$$\frac{\partial u}{\partial x^*} + \frac{\partial v}{\partial r} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x^*} + v\frac{\partial u}{\partial r} = \nu\frac{\partial}{r\partial r}\left(r\frac{\partial u}{\partial r}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x^*} + v\frac{\partial T}{\partial r} = \alpha\frac{\partial}{r\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{3}$$

where u and v are be velocity components along x^* and r directions, T be the temperature and $\alpha = k/\rho c_p$ be the thermal diffusivity of the fluid. The relevant boundary conditions for the governing problem are as follows:

$$\left.\begin{array}{l} u(r,x^*) = U(x^*), v(r,x^*) = 0, \quad T = T_w = T_\infty + AU(x^*) \text{ at } r = R \\ u(r,x^*) \to 0, \quad T = T_\infty \text{ as } r \to \infty \end{array}\right\}, \tag{4}$$

where A is a constant. Now introducing the following suitable transformation

$$x = \frac{x^*}{L}, \quad \eta = \frac{r^2 - R^2}{2Rx^*}\ Re_{x^*}^{1/2}, \quad \psi = \nu\ Re_{x^*}^{1/2} f(\eta, x),$$
$$\theta(\eta, x) = \frac{T - T_\infty}{T_w - T_\infty}. \tag{5}$$

where stream function ψ is the non-dimensional function defined through usual relationship as

$$u = \frac{\partial\psi}{r\partial r}, \quad v = -\frac{\partial\psi}{r\partial x}. \tag{6}$$

which satisfies the continuity Eq. (1). Upon using Eqs. (5) and (6) into Eqs. (2) and (3), we arrived at following transformed equations

$$\frac{r^2}{R^2}f_{\eta\eta\eta} + \frac{2x}{R\ Re_x^{1/2}}f_{\eta\eta} + \left(1 - \frac{x}{Re_x}\frac{d\ Re_x}{dx}\right)f_\eta^2 + \frac{x}{2Re_x}\frac{d\ Re_x}{dx}ff_{\eta\eta}$$
$$= x\left(f_\eta\frac{\partial f_\eta}{\partial x} - f_{\eta\eta}\frac{\partial f}{\partial x}\right), \tag{7}$$

$$\frac{1}{Pr}\left(\frac{r^2}{R^2}\theta_{\eta\eta} + \frac{2x}{R\ Re_x^{1/2}}f_{\eta\eta}\right) - x\tanh x\theta f_\eta + \frac{x}{2\ Re_x}\frac{d\ Re_x}{dx}f\theta_\eta$$
$$= x\left(f_\eta\frac{\partial\theta}{\partial x} - \theta_\eta\frac{\partial f}{\partial x}\right), \tag{8}$$

where $Re_x = U_w xL/\nu$ (local Reynolds number) and $Pr = \nu/\alpha$ (Prandtl number). Using the stretching velocity $U_w = U_0 \cosh x$ in Eqs. (7) and (8), we get

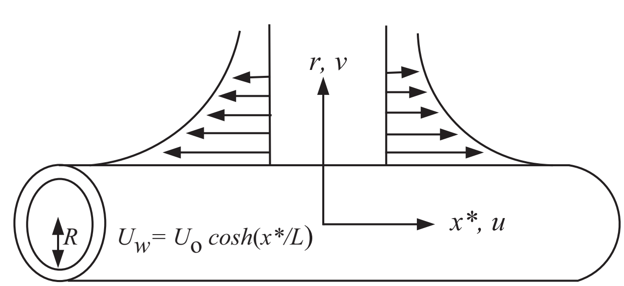

Figure 1 Geometry of the flow.