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Electron states and electron Raman scattering in an asymmetrical double quantum well: External electric field

L. Ferrer-Galindo[a], Ri. Betancourt-Riera[b], Re. Betancourt-Riera[b,*], R. Riera[c]

[a] Departamento de Física, Universidad de Sonora, Blvd. Luis Encinas y Rosales S/N, Apartado Postal 1626, Col. Centro C.P. 83000, Hermosillo, Sonora, Mexico
[b] Instituto Tecnológico de Hermosillo, Avenida Tecnológico S/N, Col. Sahuaro, C.P. 83170, Hermosillo, Sonora, Mexico
[c] Departamento de Investigación en Física, Universidad de Sonora, Blvd. Luis Encinas y Rosales S/N, Apartado Postal 5-88, Col. Centro, C.P. 83000 Hermosillo, Sonora, Mexico

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ABSTRACT

In this paper studies an electron Raman scattering process for a semiconductor, coupled and asymmetrical double quantum well. Then, the presence of an electron in a single conduction band is considered. In addition, the system is subjected to an external electric field. To carry out this study, the net Raman gain and the differential cross section are calculated. The emission spectra are interpreted and discussed. For this, we obtain the exact solutions of the electron states considering the envelope function approximation and a single parabolic conduction band, which is split into a sub-bands system due to confinement. Furthermore, the effect of the electric field on the electron states and in the differential cross section is studied. To illustrate our findings, we have considered a system growing in a $GaAs/Al_xGa_{1-x}As$ matrix.

1. Introduction

The structures based on multiple quantum wells have been studied for more than three decades because of their optical and electronic properties used in the development of applications such as solar cells, high efficiency multiband detectors and lasers [1–3]. Furthermore, it has been proposed to use them as a source of coherent mid-infrared radiation based on electron Raman scattering [4–9]. Recently, the use of Raman scattering as a control method in the growth of multiple quantum well systems has gained interest [10]. Thus, Raman scattering is a highly efficient tool, both theoretically and experimentally, for research of electronic and optical properties of low dimensional semiconductor structures [11–23].

GaAs-based semiconductors have a low macroscopic polarization, unlike those based on wurtzite nitrides (as the group-III nitride semiconductor compounds) where the macroscopic polarization is strong [24]. The magnitude of the built-in electric field induced by the piezoelectricity and the spontaneous polarization in the wurtzite nitrides semiconductors is estimated in the order of MV/cm [24–28]; however, in the *GaAs*-based semiconductors this is negligible. In consequence, this implies that the quantum-confined Stark effect on devices based on *GaAs* is simpler to control than it is in nitrides. On the other hand, from previous studies, it is known that the electric field effect and the electronic and optical properties (especially in the Raman net gain and in the Raman emission spectra) are complex in nitrides [29,30].

In this document, the exact solutions of the mathematical expressions of the electron states for a semiconductor, coupled and asymmetrical double quantum well in the presence of an external electric field are obtained. Moreover, the emission spectra and the selection rules obtained when calculating the differential cross-section corresponding to an intra-band electron Raman scattering process are shown. In addition, the Raman net gain is also analyzed in a three-level system. With this system we can analyze the structures that have already been studied, such as the step-quantum well and the double quantum well [29], which allows us an easy comparison with these systems; then, this system generalizes the aforementioned structures. For a better understanding of our results, this article has been divided into several sections: In Section 2, the electron states and the Raman differential cross-section expressions are obtained. In Section 3, the results considering a system that grows in a $GaAs/Al_xGa_{1-x}As$ matrix are presented, and the effect produced by the electric field and by the change in the system parameters is determined. Finally, in Section 4, the conclusions are presented.

* Corresponding author.
E-mail addresses: liferrerg@gmail.com (L. Ferrer-Galindo), rbetancourt2012@gmail.com (R. Betancourt-Riera), rbriera@gmail.com (R. Betancourt-Riera), rriera@cifus.uson.mx (R. Riera).

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2. Model and theory

In this section, we show the solution of the Schrödinger equation in the envelope function approximation. Then, we can determine the bound states of a confined electron in a semiconductor, coupled and asymmetrical double quantum well system; with the presence of an external electric field (F), which is constant and uniform. Therefore, the Schrödinger equation takes the following form:

$$\left\{\nabla^2 + \frac{2\mu}{\hbar^2}[\mathscr{E} - V_c - |e|Fz]\right\}\Psi = 0. \tag{1}$$

This implies that there are not bound states, but quasi-bound states, with energies given by:

$$\mathscr{E}(n, k_\perp) = \mathscr{E}_z(n) + \frac{\hbar^2}{2\mu}k_\perp^2 - i\frac{\Gamma}{2},$$

being $\mathscr{E}_z(n)$ the part of the quasi-bound state energy that corresponds to the confinement and $\hbar^2k_\perp^2/2\mu$ the energy that corresponds to the free direction. $n = 1,2, \ldots,$ is the number assigned to the electron states, while Γ is the resonance width found to be positive [31].

A coupled asymmetrical double quantum well was considered with the corresponding layer thickness $-\infty/d_1/b/d_2/+\infty$. Furthermore, the presence of constant and uniform electric field F is considered. Then, the effective mass (μ) and the confinement potential (V_c), can be chosen as:

$$\mu,\ V_c = \begin{cases} \mu_1,\ V_1, & -\infty < z < 0 \\ \mu_2,\ V_2, & 0 \le z \le l_1 \\ \mu_3,\ V_3, & l_1 < z < l_2 \\ \mu_2,\ V_2, & l_2 \le z \le l_3 \\ \mu_1,\ V_1, & l_3 < z < +\infty \end{cases} \tag{2}$$

where $l_1 = d_1$, $l_2 = d_1 + b$ and $l_3 = d_1 + b + d_2$.

The wave function takes the following form:

$$\Psi(\mathbf{r}) = \frac{\exp[i\mathbf{k}_\perp\cdot\mathbf{r}_\perp]}{\sqrt{L_xL_y}}\varphi_n(z)u_0(\mathbf{r}) \tag{3}$$

where

$$\varphi_n(z) = \begin{cases} A_1\mathrm{Ci}(\eta_{1z}), & -\infty < z < 0 \\ A_2\mathrm{Ai}(\eta_{2z}) + B_2\mathrm{Bi}(\eta_{2z}), & 0 \le z \le l_1 \\ A_3\mathrm{Ai}(\eta_{3z}) + B_3\mathrm{Bi}(\eta_{3z}), & l_1 < z < l_2 \\ A_4\mathrm{Ai}(\eta_{2z}) + B_4\mathrm{Bi}(\eta_{2z}), & l_2 \le z \le l_3 \\ A_5\mathrm{Ai}(\eta_{1z}), & l_3 < z < +\infty \end{cases}$$

$$\mathrm{Ci}(\eta) = \mathrm{Bi}(\eta) + i\mathrm{Ai}(\eta) \quad \text{and} \quad \eta_{jz} = -[2\mu_j/(eF\hbar)^2]^{1/3}[\mathscr{E}_z(n) - V_c(z) - |e|Fz].$$

Being $u_0(\mathbf{r})$ the periodic part of the Bloch function, Ai and Bi the Airy's functions and finally $j = 1,2$ or 3. Once the boundary conditions are applied, the continuity of the function Ψ and the current $(1/\mu)(\partial\Psi/\partial z)$ at the interface, $\mathscr{E}_z(n)$ are determined by the following secular equation:

$$[\Pi_{12}\Sigma_{23}^2(l_1) - \Pi_{22}\Xi_{23}^2(l_1)][\Sigma_{32}^3(l_2)\Phi_{21}^4(l_3) - \Phi_{32}^3(l_2)\Lambda_{21}^4(l_3)] = [\Pi_{12}\Phi_{23}^2(l_1) - \Pi_{22}\Lambda_{23}^2(l_1)][\Xi_{32}^3(l_2)\Phi_{21}^4(l_3) - \Lambda_{32}^3(l_2)\Lambda_{21}^4(l_3)]$$

while the constants A and B have the following form:

$$A_1 = [I_1 + I_2 + I_3 + I_4 + I_5]^{-1/2},$$
$$A_2 = \overline{A}_2A_1,\quad B_2 = \overline{B}_2A_1,$$
$$A_3 = \overline{A}_3A_1,\quad B_3 = \overline{B}_3A_1,$$
$$A_4 = \overline{A}_4A_1,\quad B_4 = \overline{B}_4A_1,$$
$$A_5 = \overline{A}_5A_1,$$

where

$$\overline{A}_2 = \frac{\Pi_{12}}{Q_2(0)},\qquad \overline{B}_2 = -\frac{\Pi_{22}}{Q_2(0)},$$
$$\overline{A}_3 = \frac{\Sigma_{23}^2(l_1)\overline{A}_2 + \Xi_{23}^2(l_1)\overline{B}_2}{Q_3(l_1)},\quad \overline{B}_3 = -\frac{\Phi_{23}^2(l_1)\overline{A}_2 + \Lambda_{23}^2(l_1)\overline{B}_2}{Q_3(l_1)},$$
$$\overline{A}_4 = \frac{\Sigma_{32}^3(l_2)\overline{A}_3 + \Xi_{32}^3(l_2)\overline{B}_3}{Q_2(l_2)},\quad \overline{B}_4 = -\frac{\Phi_{32}^3(l_2)A_3 + \Lambda_{32}^3(l_2)\overline{B}_3}{Q_2(l_2)},$$
$$\overline{A}_5 = \frac{\mathrm{Ai}(\eta_{2l_3})\overline{A}_4 + \mathrm{Bi}(\eta_{2l_3})\overline{B}_4}{\mathrm{Ai}(\eta_{1l_3})},$$

$$\Pi_{12} = \mathrm{Ci}(\eta_{10})\mathrm{Bi}'(\eta_{20}) - \alpha_1\mathrm{Ci}'(\eta_{10})\mathrm{Bi}(\eta_{20}),$$
$$\Pi_{22} = \mathrm{Ci}(\eta_{10})\mathrm{Ai}'(\eta_{20}) - \alpha_1\mathrm{Ci}'(\eta_{10})\mathrm{Ai}(\eta_{20}),$$

$$\Phi_{ij}^k(z) = \mathrm{Ai}(\eta_{iz})\mathrm{Ai}'(\eta_{jz}) - \alpha_k\mathrm{Ai}'(\eta_{iz})\mathrm{Ai}(\eta_{jz}),$$
$$\Lambda_{ij}^k(z) = \mathrm{Bi}(\eta_{iz})\mathrm{Ai}'(\eta_{jz}) - \alpha_k\mathrm{Bi}'(\eta_{iz})\mathrm{Ai}(\eta_{jz}),$$
$$\Sigma_{ij}^k(z) = \mathrm{Ai}(\eta_{iz})\mathrm{Bi}'(\eta_{jz}) - \alpha_k\mathrm{Ai}'(\eta_{iz})\mathrm{Bi}(\eta_{jz}),$$
$$\Xi_{ij}^k(z) = \mathrm{Bi}(\eta_{iz})\mathrm{Bi}'(\eta_{jz}) - \alpha_k\mathrm{Bi}'(\eta_{iz})\mathrm{Bi}(\eta_{jz}),$$

$$Q_i(z) = \mathrm{Ai}(\eta_{iz})\mathrm{Bi}'(\eta_{iz}) - \mathrm{Ai}'(\eta_{iz})\mathrm{Bi}(\eta_{iz}),$$

$$I_1 = \int_{-\infty}^0 \mathrm{Ci}^*(\eta_{1z})\mathrm{Ci}(\eta_{1z})dz,$$
$$I_2 = \int_0^{l_1} [\overline{A}_2\mathrm{Ai}(\eta_{2z}) + \overline{B}_2\mathrm{Bi}(\eta_{2z})]^*[\overline{A}_2\mathrm{Ai}(\eta_{2z}) + \overline{B}_2\mathrm{Bi}(\eta_{2z})]\,dz,$$
$$I_3 = \int_{l_1}^{l_2} [\overline{A}_3\mathrm{Ai}(\eta_{3z}) + \overline{B}_3\mathrm{Bi}(\eta_{3z})]^*[\overline{A}_3\mathrm{Ai}(\eta_{3z}) + \overline{B}_3\mathrm{Bi}(\eta_{3z})]\,dz,$$
$$I_4 = \int_{l_2}^{l_3} [\overline{A}_4\mathrm{Ai}(\eta_{2z}) + \overline{B}_4\mathrm{Bi}(\eta_{2z})]^*[\overline{A}_4\mathrm{Ai}(\eta_{2z}) + \overline{B}_4\mathrm{Bi}(\eta_{2z})]dz$$
$$\text{and}\quad I_5 = \overline{A}_5^*\overline{A}_5\int_{l_3}^{+\infty} \mathrm{Ai}^*(\eta_{1z})\mathrm{Ai}(\eta_{1z})dz,$$

with

$$\alpha_1 = [\mu_2/\mu_1]^{2/3},\quad \alpha_2 = [\mu_3/\mu_2]^{2/3},\quad \alpha_3 = [\mu_2/\mu_3]^{2/3} \quad\text{and}\quad \alpha_4 = [\mu_1/\mu_2]^{2/3}.$$

On the other hand, the differential cross-section for an intra-band electron Raman scattering process in a volume per unit solid angle (Ω) of incoming light with energy ($\hbar\omega_l$) and scattering light with energy ($\hbar\omega_s$), is calculated.

In Ref. [32] the calculation process to obtain the differential cross section is described; therefore

$$\frac{\partial^2\sigma}{\partial\omega_s\partial\Omega} = \sigma_0\Gamma_f^2\frac{\omega_s}{\omega_l}\sum_{n,n''}\frac{|M_f(n, n'')|^2}{[\hbar\omega_l - \hbar\omega_s + \mathscr{E}_z(n'') - \mathscr{E}_z(n)]^2 + \Gamma_f^2} \tag{4}$$

where

$$M_f(n, n'') = \frac{\hbar^2}{2\mu_0l_3^2}\sum_{n'} T(n, n')T(n', n'')\left[\frac{1}{\hbar\omega_s + \mathscr{E}_z(n) - \mathscr{E}_z(n') + i\Gamma_a} - \frac{1}{\hbar\omega_l - \mathscr{E}_z(n) + \mathscr{E}_z(n') - i\Gamma_b}\right], \tag{5}$$

$$\sigma_0 = \frac{\eta(\omega_s)}{\eta(\omega_l)}\left(\frac{e}{c}\right)^4\frac{4\hbar}{\pi\mu_0^2\Gamma_f}|(\mathbf{e}_s\cdot\mathbf{e}_z)(\mathbf{e}_l\cdot\mathbf{e}_z)|^2.$$

and in this case

$$T(n_a, n_b) = \mu_0l_3\left[\frac{1}{\mu_1}\int_{-\infty}^{0}\varphi_{n_a}^*(z)\varphi_{n_b}'(z)dz + \frac{1}{\mu_2}\int_0^{l_1}\varphi_{n_a}^*(z)\varphi_{n_b}'(z)dz + \frac{1}{\mu_3}\int_{l_1}^{l_2}\varphi_{n_a}^*(z)\varphi_{n_b}'(z)dz + \frac{1}{\mu_2}\int_{l_2}^{l_3}\varphi_{n_a}^*(z)\varphi_{n_b}'(z)dz + \frac{1}{\mu_1}\int_{l_3}^{+\infty}\varphi_{n_a}^*(z)\varphi_{n_b}'(z)dz\right] \tag{6}$$

where μ_0 is the electron free mass. $\eta(\omega_r)$ is the refraction index as a function of the radiation with frequency ω_r and polarization unit vector $\mathbf{e}_r$, where $r = s(l)$ indicates the incident (secondary) radiation and c is the speed of light in a vacuum. While Γ_a, Γ_b and Γ_f are the respective lifetimes of the intermediate and final states.