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ORIGINAL ARTICLE

# Nonlinear peristaltic motion of a Johnson–Segalman fluid in a tapered asymmetric channel

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**Abstract** The peristaltic flow analysis in a tapered asymmetric channel has been made for a Johnson–Segalman fluid. The tapered asymmetric channel is assumed to be formed due to a peristaltic wave train on the non-uniform walls having different amplitudes and phase. Two-dimensional equations of a Johnson–Segalman fluid have been simplified by treating a long wavelength and low Reynolds number approximations. The reduced equations are then solved for the stream function, axial velocity and axial pressure gradient using a regular perturbation technique. The expressions for the pressure rise, axial velocity and stream function are sketched and the reasons for the variations observed in various physical parameters are interpreted with valid theory. It has been noticed that peristaltic pumping region and free pumping decrease with an increase in non-uniform parameter and the situation is quite complimentary to the case of augmented pumping. It has also been observed that the size of the tapped bolus decreases with an increase in Weissenberg number.

## 1. Introduction

A variety of applications of a peristaltic motion in non-Newtonian fluids have led to renewed interest among the scientists and researchers. Such applications include urine transport from kidney to bladder, chime motion in the gastrointestinal tract, swallowing food through the esophagus, vasomotion of small blood vessels, movement of spermatozoa in human reproductive tract, blood pumps in dialysis and heart lung machine. Many studies are now found in the literature which includes analytical, numerical and experimental measurements [1–12] on the peristaltic flow in different geometries.

Though the earlier studies on peristaltic motion paid much attention on Newtonian fluids there are also systems involving non-Newtonian motion in which the relation between shear

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stress and shear rate appears to be nonlinear in character. In fact physiological fluids frequently fall under non-Newtonian category. It is quite appropriate to observe the dynamics of the fluids by taking their non-Newtonian behavior into deliberation. Only partial information on the peristaltic transport of non-Newtonian fluids is available in the literatures [13–18]. Some constitutive formulation including the Johnson–Segalman model and Giesekus model [19,20] contains a nonmonotone shear-stress–shear-rate curve. In the present problem, we focus on the Johnson–Segalman fluid. Johnson and Seglman [21] begin with a constitutive equation that is derived systematically from both the molecular theory of Gaussian networks, and the molecular bead-spring model with Hookean springs. Johnson–Segalman fluid is considered as one among the subclass of viscoelastic fluids. This model has an ability to explain the "spurt" phenomenon. The term "spurt" has been used for the description of large increase in the volume to a small increase in the driving pressure gradient. Researchers [22–25] often deployed this model to explain the spurt phenomenon. Several experimentalists have correlated the spurt with a slip at a wall. The three distinct flows of the Johnson–Segalman type have been extensively studied by Rao and Rajagopal [26].

The quest of getting accurate methods for solving resulted nonlinear models involving higher order is of utmost concern for many researchers in this field today. Various analytical methods have been put to use successfully to obtain solutions of classical nonlinear differential equations such as the method of Perturbation method, Adomian decomposition method, Differential transform method and Homotopy perturbation method, [27–31]. Hajmohammadi et al. [32] proposed based on semi-analytical methods to solve the conjugate heat transfer problems. The two semi analytical algorithms, Differential Transform Method (DTM) and Adomian Decomposition Method (ADM) are examined for solving a characteristic value problem occurring in linear stability analysis by Hajmohammadi and Nourazar [33]. The same authors discussed that the semi-analytical solution for the nonlinear integro-differential equation occurring in the problem is handled easily and accurately by implementing the Differential Transform Method (DTM) [34].

Physiologists have also observed that the intrauterine fluid flow due to myometrial contractions is peristaltic-type motion and the myometrial contractions may occur in both symmetric and asymmetric directions, De Vries et al. [35]. Eytan et al. [36] had looked at the characterization of non-pregnant woman's uterine contractions is extremely complicated as they are composed of variable amplitudes, a range of frequencies and different wavelengths. The flows in an asymmetric channel generated by peristaltic waves propagating on the walls with different amplitudes and phase have been observed by Mishra and Rao [37]. In general, asymmetric wall oscillation in non-uniform channels may also exist in biological conduits, e.g., the uterus [38–41]. Maxey and Riley [42] disused the particle motion under non-uniform flow and analytically derived an equation for the particle motion through which many forces are measured.

The peristaltic transport of Johnson–Segalman fluid under a magnetic field has been studied by Elshahed and Haroun [43]. Nadeem and Akbar [44] have carried out a detailed analysis on the peristaltic flow of a Johnson–Segalman fluid in a non-uniform tube. The effect of magnetic field on the motion of a Johnson–Segalman fluid has been sculpted by Hayat and Ali [45]. The peristaltic transport of MHD Johnson–Segalman fluid in a planar channel has been discussed by Hayat et al. [46]. Peristaltic motion of Johnson–Segalman fluid in an asymmetric and planar channel was also reported by Hayat et al. [47]. Nadeem and Akbar [48] have made significant study on the peristaltic flow of Johnson–Segalman fluid in a non-uniform tube with heat transfer. Numerous theoretical works have been performed by many authors to investigate the various effects related to the peristaltic transport of a non-Newtonian fluid [49–54].

Besides on the foregoing works, we are interested to study the effect of a Johnson–Segalman fluid in the peristaltic wave train on the non-uniform walls of having different amplitudes and phase. It is worthwhile to note that such analysis does not seem to be made available in the existing literature. The governing equations have been simplified by presuming long wavelength approximation and low Reynolds number. With the aid of regular perturbation method, asymptotic solutions for stream function and pressure gradient were obtained. The expressions for pressure rise per wave length were also calculated. The effects of different physical parameters appearing in the problem have been discussed in lucid manner and shown graphically for better understanding.

## 2. Governing equations

Consider an incompressible, Johnson–Segalman fluid confined in a two dimensional infinite asymmetric channel. We employ a rectangular coordinate system with $X$ parallel and $Y$ normal to the channel walls. Moreover, we consider an infinite wave train traveling with velocity c along the channel walls. The asymmetry in the channel is induced by assuming the non-uniform peristaltic wave train on the walls to have different amplitudes and phases Fig. 1. The shape of the channel walls is

$$H_2(X,t') = d + k'X + a_2 \sin\left(\frac{2\pi X}{\lambda} - \frac{ct'}{\lambda}\right), \quad \ldots \text{ upper wall} \tag{1}$$

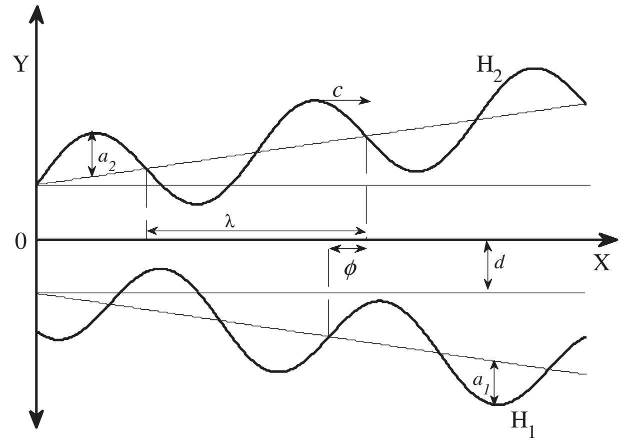

**Figure 1** Schematic diagram of a tapered asymmetric channel.