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Investigation on negative hybrid-resonant bands of elastic metamaterials by revised effective medium theory

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ABSTRACT

In this paper, a revised effective medium theory is presented to investigate negative dispersion bands which are formed by the hybridization effect of multiple resonances. The negative hybrid-resonant bands are selective for propagation of compressional or shear waves. With a simplified model, a new negative dispersion band is firstly achieved by combinations of negative transverse modulus and negative mass density, thus three negative dispersion bands are formed by hybridization effect of a monopolar or quadrupolar resonance and a dipolar resonance. Secondly, transmission spectrums are also performed to demonstrate the negative behaviors of three hybrid resonant bands and illustrate their unusual transmission properties. Moreover, a revised effective medium theory based on numerical simulation is derived to obtain four effective parameters of the elastic metamaterials along two wave vectors when the long wavelength limitation is not fully satisfied, which perfectly explain the unusual transmissions of negative dispersion bands. Finally, the physical mechanisms of negative parameters are systematically investigated by three typical deformation states. The work has guiding significances for designing the elastic metamaterials structures with negative hybrid-resonant bands.

1. Introduction

Metamaterials belong to man-made composite media structured on a scale much smaller than a wavelength and its building block can exhibit a resonance under wave excitation [1], which is developed from the concept of negative refractive index in classical waves [2–4]. In 2000, Liu et al. fabricated the first resonant structure consisting of rubber-coated lead spheres embedded in an epoxy matrix and observed two bandgaps in the long-wavelength regime, wherein the bandgap was attributed to the negative mass density (NMD) [5]. In 2006, Fang et al. experimentally observed the negative bulk modulus (NBM) in a type of ultrasonic metamaterial that consisted of an array of subwavelength Helmholtz resonators with designed acoustic inductance and capacitance [6]. Furthermore, researchers considered various methods to obtain dynamic negative parameters in acoustic and elastic metamaterials [7–16]. In 2004, Li et al. realized simultaneously NMD and NBM by dispersing soft rubber in water [7]. In 2010, Lee et al. fabricated an acoustic composite structure by a periodic array of interspaced membranes (support negative density) and side holes (support negative modulus) [11]. Ding et al. designed a double unit structure, in which one unit had very strong monopolar resonance while the other one had very strong dipolar resonance, this was a realization of NBM and NMD in a solid material [9]. In 2013, Yang et al. used coupled membranes to design a simple acoustic double negative metamaterial that could produce simultaneously monopolar and dipolar resonances [14]. Moreover, Liu et al. [13] and Wang et al. [15,16] achieved NMD and NBM in elastic metamaterials with chiral microstructures, wherein the unit cells could produce simultaneous translational and rotational resonances. Wu et al. designed a structure constitute a silicone rubber filled with water, which theoretically showed that both the mass density and shear modulus (NSM) were negative [17]; Lai et al. designed a type of elastic metamaterial with its unit cell comprising a multi-mass locally resonant inclusion, which could exhibit multiple resonances including monopolar, dipolar and quadrupolar resonances [18]. They used four effective parameters$\{\rho^{eff}, \kappa^{eff}, \mu^{eff}, c_{44}^{eff}\}$to describe elastic metamaterials, and proved negative dispersion bands were formed by combinations of NBM or NSM and NMD. Here, we have a question that can a negative dispersion band be achieved by combinations of negative transverse modulus (NTMc_{44}^{eff}) and NMD? These acoustic metamaterials with negative effective parameters are used to control sound propagation, which could generate special acoustic phenomenon including sound insulation and absorption [19–22], sound focusing [23,24], sound imaging [25,26] and acoustic cloak [27–30] et al.

The effective medium theory is the core of the study of elastic and

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acoustic metamaterials, which could give quantitative descriptions and detailed solutions for dynamic effective parameters. The effective medium theory based on the coherent-potential approximation has been used to predict effective parameters of periodic structures [31]. Wu et al. developed a theory to derive the effective parameters of two-dimensional elastic metamaterials that consisted of cylindrical scatters embedded in a solid matrix by the scattering properties of the scatters [32,33]. Zhou et al. used an analytic model to derive the effective material parameters by averaging physical fields from local constituents, which was based on the Mie scattering solution [34]. These methods above are theoretical derivations to calculate effective parameters for two-dimensional (2D) elastic metamaterials. However, developing a simple numerical method to determine the effective properties is a huge requirement. In 2007, Fokin et al. [35] developed a method that used reflection and transmission coefficients in acoustic field to extract effective parameters, the advantage of the retrieval method was the ability to analyze various acoustic metamaterials, whereas the disadvantage was only effective mass density and bulk modulus could be calculated. In 2012, Zhu et al. represented a numerical approach to calculate the effective parameters of anisotropic metamaterial plate, which was also a leading work for us to explore the effective properties by numerical simulation [36]. Many researchers have also discussed the high frequency or non-local behavior of the effective parameters with rigorous arguments [37–39]. We want to derive a revised effective medium theory based on numerical simulation to deal with a problem, which is how to obtain effective parameters of anisotropic elastic metamaterials along different wave vectors when the long wavelength assumption is not fully satisfied.

In this paper, we propose a revised effective medium theory to explain the negative hybrid-resonant bands of elastic metamaterials. Compared with the work presented by Lai et al., we present a simplified model to achieve three negative hybrid-resonant bands, and one of them is achieved by combinations of NTM and NMD. We perform transmission spectrum to analyze the negative dispersion bands and illustrate their unusual transmission properties. We also derive a revised effective medium theory based on numerical simulation to obtain effective parameters of the elastic metamaterials along two wave vectors when the long wavelength limitation is not fully satisfied. The calculated effective parameters can perfectly predict transmissions of the negative dispersive bands. Finally, the physical mechanisms of negative effective parameters are systematically investigated by three typical deformation states.

2. The negative hybrid-resonant bands within elastic metamaterials

A unit cell of the proposed elastic metamaterial is shown in Fig. 1(a), wherein four rectangle steel blocks are embedded in a square foam matrix. The lattice constant of the square cell is 100 mm, the side lengths of the rectangle steel blocks are 24 mm and 16 mm respectively, and the steel blocks locate at a distance of 24 mm from the center. The mass density and two Lame's constant are$\rho_f = 115$ kg/m^3 $\lambda_f = 6 \times 10^6$ N/m^2, $\mu_f = 3 \times 10^6$ N/m^2, respectively for the foam; and those of steel are $\rho_s = 7.9 \times 10^3$ kg/m^3, $\lambda_s = 1.11 \times 10^{11}$ N/m^2, $\mu_s = 8.28 \times 10^{10}$ N/m^2,respectively. Compared with the ref. 18, rubber materials are not used as the springs in this study. However, the four rectangle steel blocks are still considered as masses. At certain frequency, multiple resonances are generated by synergic motions of masses, and collective motion could produce the dipolar resonance (for negative mass density), whereas relative motions could produce the monopolar and quadrupolar resonances (for negative moduli).

2.1. The mechanism of negative hybrid-resonant bands

We calculate band structure and eigenstates by the finite element software COMSOL Multiphysics to discover the unusual propagation property of the elastic metamaterial. The band structure is shown in Fig. 1(b), the curvatures of three orange dispersion bands are negative. The bandwidth of the orange band-1 is about 140 Hz (405–545 Hz), that of the orange band-2 is about 76 Hz (664–740 Hz) and that of the orange band-3 is about 37 Hz (817–854 Hz). The cyan regions denote the frequency range of NMD, which are produced by the collective motions of four steel blocks. In the interested frequency range, the kinetic energy (both vibrational and rotational motions) mostly concentrate in the steel blocks, so it means the motions of steel blocks are the cause that produces negative dispersion bands. We note that around 800 Hz,the transverse and longitudinal wavelengths in the foam matrix are about 20 cm and 40 cm respectively. The lattice constant is 10 cm, so it has the same order of magnitude with wavelength in interested frequency range. In order to explain the formation mechanism of negative dispersion bands, an investigation of the eigenstates is needed to give us a clear picture of the physical origin of the bands.

Because the collective motions of steel blocks could support dipolar resonances and further produce NMD, while the relative motions support monopolar or quadrupolar resonances and further produce NBM or NSM, we could investigate the eigenstates in the hybridized bands to reveal the physical mechanism of the negative dispersion bands. Fig. 2 shows some eigenstates in negative dispersion bands, where the color represents the amplitude of displacement (blue/red for small/large values) and cones indicate the displacement vectors. Fig. 2(a)–(c) respectively shows the eigenstates of three high symmetric points (A1-A3) in the orange band-1. The up and down steel blocks move to the center, while the left and right ones move away from the center, and it is clearly that these eigenstates are quadrupolar resonances. The quadrupolar resonance produce NSM, so A1/A3 are the starting points and A2 is the end point of the NSM in the $\Gamma M/\ \Gamma X$direction. The orange

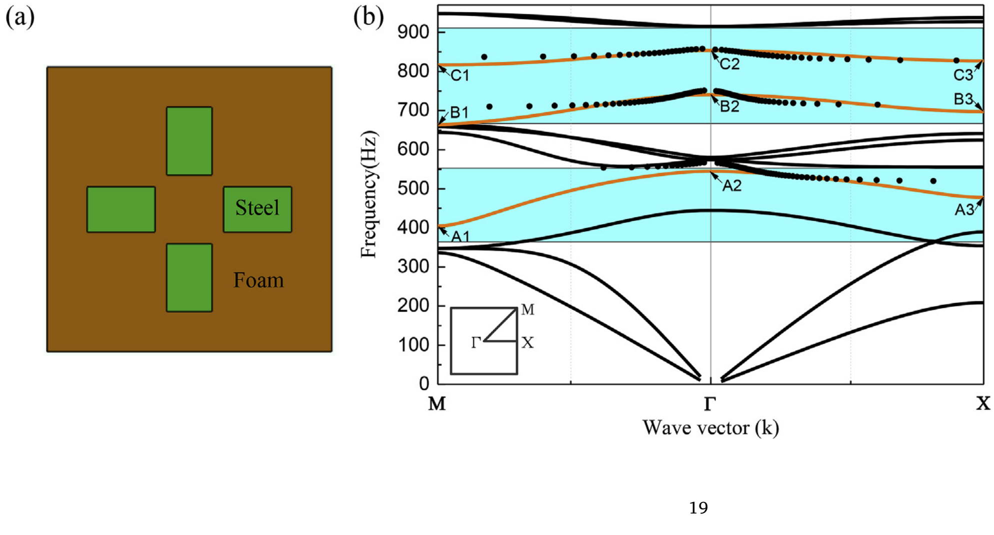

Fig. 1. The geometric model and band structure of the unit cell. (a) The geometric model consisted of four rectangle steel blocks embedded in a square foam matrix. (b) Three orange lines denote negative dispersion bands. The dots denote the dispersion bands obtained by using the revised effective medium theory. The cyan regions represent the frequency ranges of negative mass density. The inset is Brillouin zone of the unit cell. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)