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ORIGINAL ARTICLE

Exact solutions to the (3 + 1)-dimensional coupled Klein–Gordon–Zakharov equation using exp(−Φ(ξ))-expansion method

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KEYWORDS

The nonlinear coupled Klein–Gordon–Zakharov equation;
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Solitary wave solutions;
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Abstract In this article, the exp(−Φ(ξ))-expansion method is modified for (3 + 1)-dimensional space–time coordinate system and successfully implemented to construct the new exact traveling wave solutions of the (3 + 1)-dimensional coupled Klein–Gordon–Zakharov equation. The solutions of this equation are expressed in terms of hyperbolic, trigonometric, exponential and rational functions. The results illustrate its effectiveness for solving nonlinear coupled partial differential equations arises in mathematical physics and engineering. The annihilation phenomena of the wave propagation in the *x*–*y* plane are also investigated. Furthermore, the three-dimensional surface plots due to the obtained solutions are also given to make the dynamics of the equation visible.

1. Introduction

It is well known that partial differential equations (PDEs) are special kind of nonlinear evolution equations. Already a large number of authors [1–3] have derived the nonlinear PDEs considering different theoretical model equations. Many issues of physical phenomena in nature may be described by solving the nonlinear PDEs. The explicit solutions of these equations play a vital role to study such physical phenomena in mathematical physics. Thus, the nonlinear equations may become most exciting and enormously active research areas for mathematicians, physicists and engineers. On the other hand, the traveling wave solutions play an essential role to study the models arising from various natural and complex phenomena in the field of applied sciences and engineering. For instance, the wave propagation phenomena were observed in optical fibers, elastic media, quantum mechanics, fluids dynamics, chemical physics, solid mechanics, biophysics, Higgs mechanism, quantum field theory, plasma physics and so on. To obtain the exact solutions of the nonlinear PDEs, the initial dominant mathematical tool was the inverse scattering method [4]. Some of other most efficient methods for finding analytical solutions of the nonlinear PDEs are the three-wave method [5], the improved F-expansion method [6], the generalized method [7], the Weiss–Tabor–Carnevale method [8], the tanh-function method [9], the extended tanh-method [10,11], the modified extended tanh-function method [12,13], the Jacobi elliptic function expansion method [14], the extended Jacobi elliptic function expansion rational method [15], the Exp-function method [16–18], the truncated Painleve expansion method [19], the homogeneous balance method [20–22] and so on. Recently, the exp(−Φ(ξ))-expansion method [23–27] has also been used

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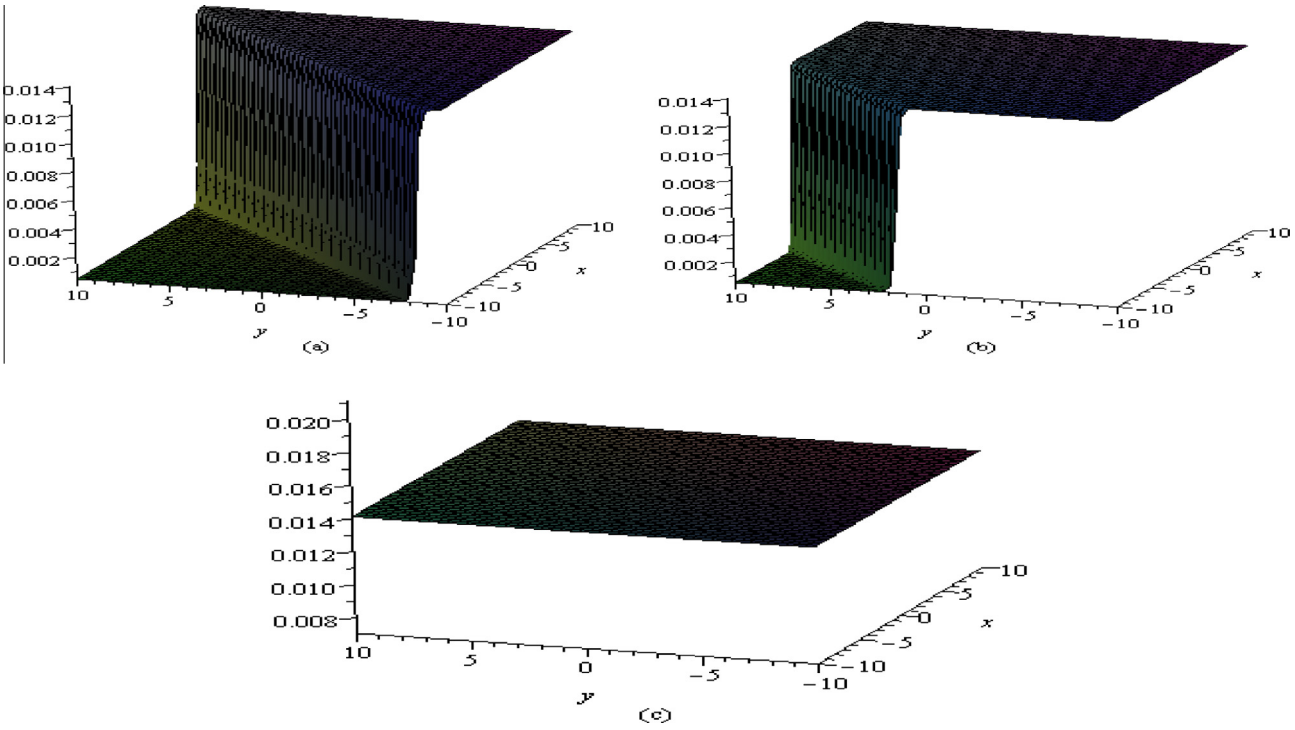

Figure 1 Shape of $|u_1(x,y)|$ in x–y plane at (a) $t = 0$, (b) $t = 10$ and (c) $t = 30$.

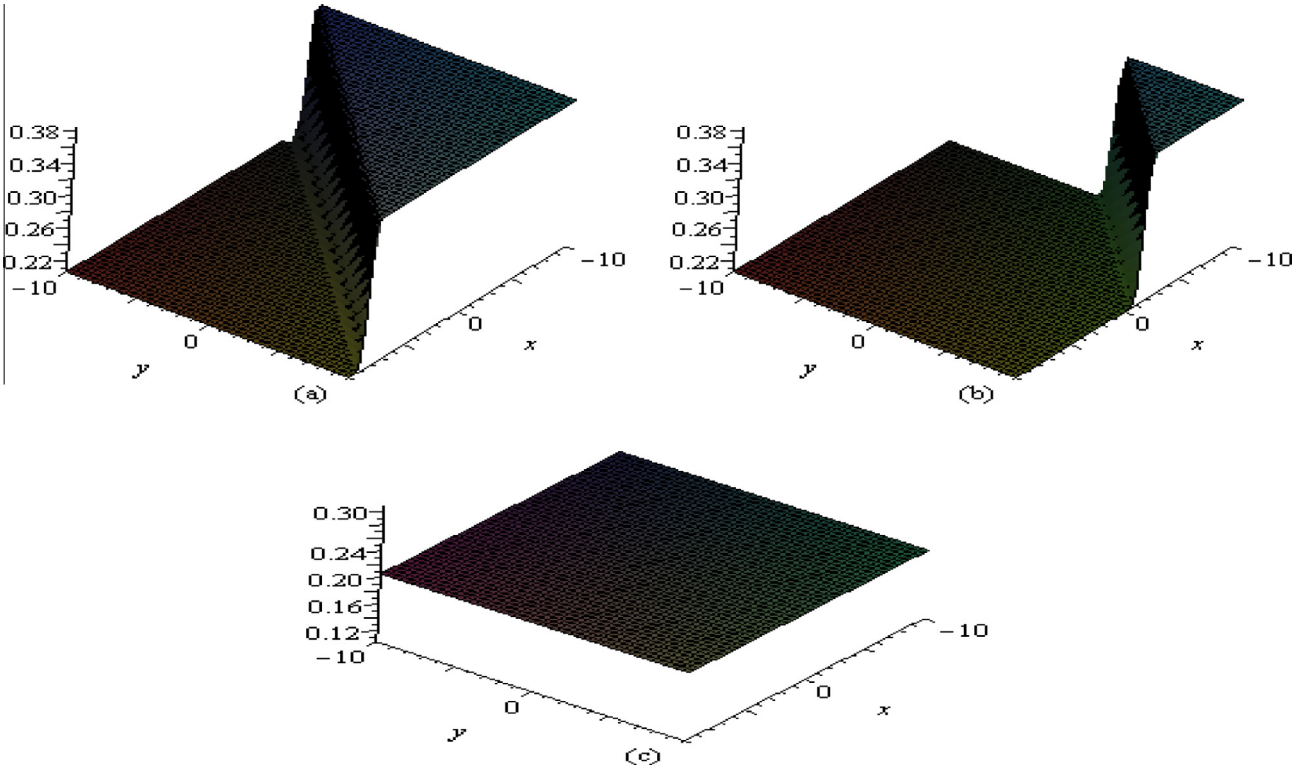

Figure 2 Shape of v_1 in x–y plane at (a) $t = 0$, (b) $t = 10$ and (c) $t = 30$.

to find the exact traveling wave solutions of single nonlinear PDEs. Moreover, many researchers [28–32] have investigated the exact solutions of the coupled nonlinear PDEs, that is, the Maccari system, Higgs field equation, $(1+1)$-dimensional nonlinear coupled Klein–Gordon–Zakharov equation, etc. employing the Exp-function method [28], truncated expansion

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