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ORIGINAL ARTICLE

New approach method for solving Duffing-type nonlinear oscillator

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Abstract In this paper attempts have been done to solve nonlinear oscillator by using Akbari–Ganji's Method (AGM). Solving nonlinear equation is difficult due to its high nonlinearity. This new approach is emerged after comparing the achieved solutions with numerical method and exact solution.

Results are presented for different values of amplitude vibration of the problem parameters which would certainly illustrate that this method (AGM) is efficient and has enough accuracy in comparison with other semi analytical and numerical methods. Moreover, results demonstrate that AGM could be applicable through other methods in nonlinear problems with high nonlinearity. Furthermore, convergence problems for solving nonlinear equations by using AGM appear small.

1. Introduction

Nonlinear oscillator models have been widely used in many areas of physics and engineering and are of significant importance in mechanical and structural dynamics for the comprehensive understanding and accurate prediction of motion, and in this investigation attempts have been made to solve nonlinear oscillator which has high nonlinearity.

Consider a nonlinear oscillator modeled by the following governing nonlinear differential equation [1]:

$$\frac{d^2}{dt^2}\theta(t) + \frac{\theta^3(t)}{1+\theta^2(t)} = 0 \tag{1}$$

With the following initial conditions:

$$\theta(0) = A, \quad \frac{d}{dt}\theta(0) = 0 \tag{2}$$

For small values of parameter θ, the governing Eq. (1) is that a Duffing-type nonlinear oscillator, i.e. $\frac{d^2}{dt^2}\theta(t) + \theta^3(t) \cong 0$, while for large values of θ the equation approximates that of a linear harmonic oscillator, i.e. $\frac{d^2}{dt^2}\theta(t) + \theta(t) \cong 0$. Hence, Eq. (1) is called the Duffing-harmonic oscillator [1].

Oscillators are used in different fields of engineering; therefore, using simple procedure for solving the governing nonlinear equation of them is considerable from decades and many researchers trying to reach acceptable solution for these equations due to their nonlinearity by utilizing analytical and semi-analytical methods such as: Homotopy Analysis Method [4–5], the He's Amplitude Frequency Formulation (HAFF) method [6,7], Parameter-Expansion Method [8], Energy Bal-

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Nomenclature

AGM	Akbari–Ganji's Method	ω_0	angular frequency
τ	torque	$\theta(t)$	angular displacement
M	mass properties	$\dot{\theta}(t)$	angular velocity
A	vibration amplitude		

ance Method [9–10], Differential Transformation Method (DTM) [11], Homotopy Perturbation Method [12–15], Adomian Decomposition Method [16–18], EXP-function Method [19–22] and Variational Iteration Method [23–26].

To obtain an accurate analytical solution for frequency–amplitude relation of the Duffing-harmonic oscillator in different range of amplitude vibration due to different usage of this oscillator, this paper employs Akbari–Ganji's Method (AGM) [2,3] which is a powerful and accurate method for solving nonlinear equations with respect to its simplicity through other semi analytical methods. Through these procedures with respect to the basic idea of the method a trial function would assume as solution of mentioned nonlinear problem then by set of algebraic calculation the constant parameters of trial function such as angular frequency would be obtained. Solving process of AGM for this problem has been done for different amounts of amplitude vibration with various terms for trial functions to present that AGM is applicable for solving nonlinear equations with high nonlinearity and by comparing with numerical solution it would be obvious that this method has enough efficiency and simplicity.

The main purpose of AGM is obtaining the accurate solution with simple algebraic calculation in which in comparison with other methods the process would be simpler and the obtained solution would be acceptable with minor errors in comparison with numerical method. It is necessary to mention that a summary of the excellence of this method in comparison with the other approaches can be considered as follows: initial conditions are needed in accordance with the order of differential equations in the solution procedure but when the number of initial conditions is less than the order of the differential equation, this approach can create additional new initial conditions in regard to the own differential equation and its derivatives. Therefore, it is logical to mention that AGM is operational for miscellaneous nonlinear differential equations in comparison with the other methods.

2. Mathematical formulation

The system under consideration is shown in Fig. 1. The rigid disk is assumed to with mass M rotating at angular velocity $\dot{\theta}$ about the inertial Z-axis. Therefore, torque τ effect on the hub causes it to rotate only. The process of formulation would be as follows:

$$M\ddot{\theta}(t) + k_t\theta(t) = \tau \tag{3}$$

By the assumption of the following values for oscillator's equation, Eq. (1) would be obtained:

$$k_t = \frac{\theta^2(t)}{1+\theta^2(t)}, \quad M = 1, \quad \tau = 0 \tag{4}$$

$$\frac{d^2}{dt^2}\theta(t) + \frac{\theta^3(t)}{1+\theta^2(t)} = 0 \tag{5}$$

With the following initial conditions:

$$\theta(0) = A, \quad \frac{d}{dt}\theta(0) = 0 \tag{6}$$

3. Basic idea of Akbari–Ganji's Method

In general, vibrational equations and their initial conditions are defined for different systems as follows:

$$f(\ddot{u}, \dot{u}, u, F_0 \sin(\omega_0 t)) = 0 \tag{7}$$

Parameter (ω_0) angular frequency of the harmonic force exerted on the system and (F_0) the maximum amplitude its.

And initial conditions are as follows:

$$\{u(t) = u_0, \quad \dot{u}(t) = 0, \quad at\ t = 0\} \tag{8}$$

3.1. Choosing the answer of the governing equation for solving differential equations by AGM

In AGM, a total answer with constant coefficients is required in order to solve differential equations in various fields of study such as vibrations, structures, fluids and heat transfer. In vibrational systems with respect to the kind of vibration, it is necessary to choose the mentioned answer in AGM. To

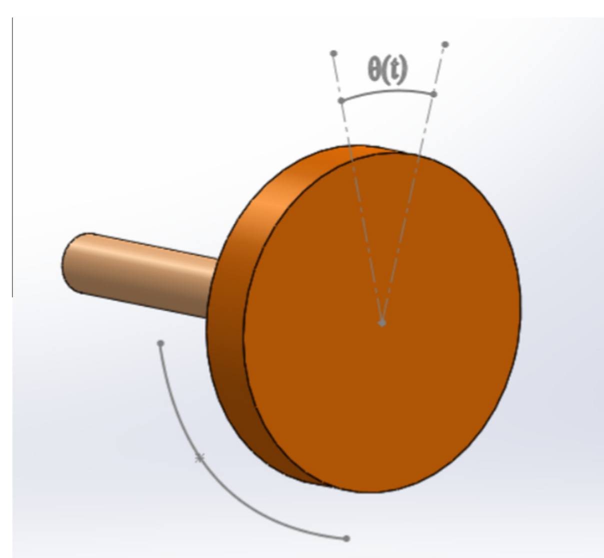

Figure 1 The geometry of the oscillator.