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Mermin-Wagner at the crossover temperature

Christoph P. Hofmann

Facultad de Ciencias, Universidad de Colima, Bernal Díaz del Castillo 340, Colima C.P. 28045, Mexico

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ABSTRACT

Mermin-Wagner excludes spontaneous (staggered) magnetization in isotropic ferromagnetic (antiferromagnetic) Heisenberg models at finite temperature in spatial dimensions $d \leq 2$. While the proof relies on the (microscopic) Bogoliubov inequality, here we illuminate the theorem from an effective field theory point of view. We estimate the crossover temperature T_c and show that, in weak external fields H, it tends to zero: $T_c \propto \sqrt{H}$ ($d = 1$) and $T_c \propto 1/|\ln H|$ ($d = 2$).

1. Introduction

In the article by Mermin and Wagner [1], absence of spontaneous symmetry breaking in isotropic Heisenberg models at finite temperature in spatial dimensions $d \leq 2$ is demonstrated by considering the magnetization or staggered magnetization in weak external magnetic or staggered fields. When the external field tends to zero, while the finite temperature is kept fixed, the (staggered) magnetization tends to zero as well. The theorem states that the functional dependence between (staggered) magnetization m and weak external field H is characterized by a power law in $d = 1$, while in $d = 2$ the connection is logarithmic,

$$m < c_1 \frac{H^{1/3}}{T^{2/3}} \qquad (d=1)\,,$$
$$m < c_2 \frac{1}{\sqrt{T}\sqrt{|\ln H|}} \qquad (d=2)\,. \tag{1.1}$$

Note that it is irrelevant whether ferromagnetic or antiferromagnetic interactions are considered.

The Mermin-Wagner theorem is based on a microscopic description of ferro- and antiferromagnets, and a crucial ingredient in its proof is the Bogoliubov inequality [2] that turned out to be very useful in different contexts. In fact, the article by Hohenberg [3] on the absence of conventional superfluid or superconducting order in $d = 1$ and $d = 2$ is also based on the Bogoliubov inequality.

Alternatively, systems exhibiting collective magnetic behavior can be analyzed within effective Lagrangian field theory – this is the method the present study is based upon. The question then arises of how the Mermin-Wagner theorem reflects itself in the effective field theory point of view, and how effective and microscopic perspectives are related to each other.

In analogy to the microscopic approach, we consider the (staggered) magnetization as a function of temperature and external field: $m(T, H)$. For a given constant field strength H, the (staggered) magnetization decreases as temperature grows and eventually becomes zero in the effective field theory description. We use the condition $m(T_c, H_c) = 0$ to estimate the crossover temperature T_c in terms of the external field. Although the effective theory operates at low temperatures, the extrapolation of the (staggered) magnetization curves to the point $m = 0$ still provides reasonable estimates for T_c.

In three spatial dimensions, T_c tends to a finite value in the limit $H \to 0$: this defines the Curie (or Néel) temperature where (anti)ferromagnetic order breaks down in a second order phase transition. Below T_c – in the absence of the external field – spontaneous magnetic order exists. Since we are in three spatial dimensions, Mermin-Wagner does not apply.

In lower spatial dimensions, however, the situation is qualitatively different: the crossover temperature T_c, estimated from effective field theory, tends to zero in the limit $H \to 0$. Accordingly, no spontaneous magnetization or spontaneous staggered magnetization can exist at finite temperatures in $d \leq 2$. This is how the Mermin-Wagner theorem manifests itself on the effective field theory level – this is our first insight.

Our second insight is that the functional dependence between T_c and H_c that we obtain from the condition $m(T_c, H_c) = 0$, remarkably, is the same as in the Mermin-Wagner inequalities (1.1): in $d = 1$ (and $d = 3$) we get a power law, $T_c \propto \sqrt{H_c}$, while in $d = 2$ we find $T_c \propto 1/|\ln H_c|$.

E-mail address: christoph.peter.hofmann@gmail.com.

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Finally, our third insight concerns universality: as in the case of the Mermin-Wagner inequalities, the functional dependence between T_c and H_c is universal: it is a consequence of the spatial dimension only and does not depend on whether ferromagnetic or antiferromagnetic order is considered.

We also explore the interplay between effective and microscopic description by deriving approximate upper bounds for the (staggered) magnetization at the estimated crossover temperatures. Apart from the cases $d = 1, 2$, we also include ferromagnets and antiferromagnets in three spatial dimensions. As it turns out, the approximate upper bounds for the (staggered) magnetization are not restrictive. Nevertheless, we briefly report our findings in an appendix.

The rest of the paper is organized as follows. In Section 2 we recapitulate the essentials of the Mermin-Wagner theorem, and then show how the theorem manifests itself in the effective field theory description. The connection between crossover temperature and external field is derived in Section 3 for ferromagnets and antiferromagnets in $d = 1$, 2, 3. Finally, Section 4 contains our conclusions. While Appendix A contains some technical details, approximate upper bounds for the (staggered) magnetization for the various systems of interest are reported in Appendix B.

2. Mermin-Wagner theorem

2.1. Rigorous statement on the microscopic level

The theorem by Mermin and Wagner [1] states that there can be no spontaneous symmetry breaking at finite temperature in the isotropic Heisenberg model,

$$\mathcal{H}_0 = -\frac{1}{2} \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j \,, \tag{2.1}$$

in spatial dimensions less or equal two. The theorem includes both ferromagnetic ($J_{ij} > 0$) and antiferromagnetic ($J_{ij} < 0$) order.

More concretely, the authors consider the quantity $m(T, H)$: the (staggered) magnetization per particle as a function of temperature and an external field H. In the case of the ferromagnet, this is the magnetic field that points into the z-direction,

$$\mathcal{H} = \mathcal{H}_0 - \sum_i S_i^z H \,, \tag{2.2}$$

while for antiferromagnetic coupling,

$$\mathcal{H} = \mathcal{H}_0 - \sum_i (-1)^i S_i^z H \,, \tag{2.3}$$

we are dealing with a staggered field.

In the limit $H \to 0$, while keeping T constant, the (staggered) magnetization tends to zero,

$$\lim_{H\to 0} m(T,H) = 0 \qquad (d \leq 2) \,. \tag{2.4}$$

Accordingly, spontaneous symmetry breaking is ruled out at finite temperature. The proof is based on the Bogoliubov inequality that leads to the explicit relations

$$\begin{aligned} m &< c_1 \frac{H^{1/3}}{T^{2/3}} \qquad (d = 1) \,, \\ m &< c_2 \frac{1}{\sqrt{T}\sqrt{|\ln H|}} \qquad (d = 2) \,, \end{aligned} \tag{2.5}$$

provided that the external field H is weak. Although the proof in the original article refers to the Heisenberg model, it can be extended to the XY model or the Hubbard model, among others (see, e.g., Refs. [4–7]). Furthermore, the analog of the Mermin-Wagner theorem that emerges in relativistic field theories was first proven and discussed by Coleman in Ref. [8].

2.2. Manifestation of Mermin-Wagner on the effective level

The systems we address in this study are ferro- and antiferromagnetic films ($d = 2$) as well as ferromagnetic spin chains ($d = 1$). We also include ferro- and antiferromagnetic crystals ($d = 3$) in order to emphasize the qualitative difference with respect to the physics in lower spatial dimensions. However, we do not consider antiferromagnetic spin chains, since they are more subtle both on the effective and microscopic level.[1]

Complementary to the microscopic description where the Mermin-Wagner proof is based upon, we use effective field theory to explore the low-temperature properties of ferro- and antiferromagnets. The method relies on the fact that the spin-waves – the collective excitations – are the relevant degrees of freedom at low temperatures.[2] The basic input we need is the dispersion relation of the spin waves in the external field. Irrespective of the spatial dimension, the leading term for ferromagnetic spin waves is quadratic,

$$\omega(\vec{k}, H) = \gamma \vec{k}^2 + H \,, \tag{2.6}$$

while the leading term for antiferromagnetic spin waves takes the relativistic form

$$\omega(\vec{k}, H) = \sqrt{v^2 \vec{k}^2 + \gamma_s H} \,. \tag{2.7}$$

The external field $\vec{H} = (0, 0, H)$ is aligned with the (staggered) magnetization vector $\vec{m}(T, H) = (0, 0, m(T, H))$. The constants γ, γ_s and v depend on the microscopic parameters S (spin quantum number), J (exchange integral), and a (lattice constant), as well as on the geometry of the system. Below, in the formulas for the (staggered) magnetization, we express the constants γ, γ_s, v in terms of microscopic parameters for each system under consideration.

With the dispersion relation we calculate the free energy density at one-loop order as

$$z = z_0 + n \frac{T}{(2\pi)^d} \int d^d k \ln\left(1 - e^{-\omega(\vec{k},H)/T}\right) \,, \tag{2.8}$$

where z_0 is the energy density of the vacuum and n is the number of independent spin-wave excitations: in ferromagnets we have $n = 1$, in antiferromagnets we have $n = 2$. The (staggered) magnetization is then obtained via

$$m(T, H) = -\frac{\partial z(T, H)}{\partial H} \,. \tag{2.9}$$

For the various systems of interest we have [11–15].

- Ferromagnetic spin chains

$$m(T, H) = S - \frac{1}{2\pi^{1/2} J^{1/2} S^{1/2}} T^{1/2} \sum_{n=1}^{\infty} \frac{e^{-nH/T}}{n^{1/2}} \,. \tag{2.10}$$

- Ferromagnetic films

$$m(T, H) = S - \frac{1}{4\pi J S} T \sum_{n=1}^{\infty} \frac{e^{-nH/T}}{n} \,. \tag{2.11}$$

- Simple cubic ferromagnetic crystals

$$m(T, H) = S - \frac{1}{8\pi^{3/2} J^{3/2} S^{3/2}} T^{3/2} \sum_{n=1}^{\infty} \frac{e^{-nH/T}}{n^{3/2}} \,. \tag{2.12}$$

- Antiferromagnetic films

$$m(T, H) = m_0 + \frac{m_0^{3/2} v}{8\pi \rho^{3/2}} \sqrt{H} + \frac{m_0}{2\pi\rho} T \ln\left[1 - \exp\left(-\frac{v m_0^{1/2}}{\rho^{1/2}} \frac{\sqrt{H}}{T}\right)\right] \,. \tag{2.13}$$

[1] Antiferromagnetic spin chains will be analyzed elsewhere.

[2] Pedagogical outlines of the effective Lagrangian method with applications to condensed matter systems are, e.g., Refs. [9,10].

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