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# A new solution for nonlinear Dual Phase Lagging heat conduction problem

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**Abstract** The application of new methods to solution of non-Fourier heat transfer problems has always been one of the interesting topics among thermal science researchers. In this paper, the effect of laser, as a heat source, on a thin film was studied. The Dual Phase Lagging (DPL) non-Fourier heat conduction model was used for thermal analysis. The thermal conductivity was assumed temperature-dependent which resulted in a nonlinear equation. The obtained equations were solved using the approximate-analytical Adomian Decomposition Method (ADM). It was concluded that the nonlinear analysis is important in non-Fourier heat conduction problems. Significant differences were observed between the Fourier and non-Fourier solutions which stress the importance of non-Fourier solutions in the similar problems.


## 1. Introduction

Heat conduction is a mechanism of heat transfer during which thermal energy is transferred from an area with higher temperature to an area with lower temperature. A fundamental equation that can describe the mentioned mechanism well was first introduced in 1822 by a French physicist called Joseph Fourier in a thesis entitled "analytical theory of heat" [1]. Parabolic classic equation of Fourier heat conduction was used until 1950 in all analyses at that time, while all scientists accepted that hypothesis of this equation based on infinite motion speed of thermal energy inside matter was a non-physical hypothesis. Of course, this hypothesis is valid in many conventional applications but Fourier law was not able to predict correctly thermal behavior of the matter in some cases such as heat transfer at very low temperatures [2], heat transfer in very small sizes [3] or heat transfer with very high rate in short times [4].

Cattaneo [5] and Vernotte [6] presented a modified model for heat conduction in independent studies. The model was named after these two scientists as the Cattaneo–Vernotte (C–V) model. Their model accounts for the inertia caused by the acceleration of heat flux and, thus, resolves the paradox present in the Fourier model. Fourier model, due to its parabolic nature, states that thermal disturbances propagate in the body with an infinite velocity. This problem was resolved in the C–V non-Fourier model, where a specific velocity was considered for the propagation of thermal disturbances. The C–V model states that in the case of occurrence of a tempera-

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**Nomenclature**

| | |
|---|---|
| $c_0$ | reference speed of thermal wave (m s$^{-1}$) |
| $c_p$ | specific heat (J kg$^{-1}$ K$^{-1}$) |
| $FO$ | Fourier number |
| $g$ | heat source (W m$^{-3}$) |
| $\tilde{g}$ | dimensionless heat source |
| $g_l$ | dimensionless energy magnitude factor at the left boundary |
| $g_r$ | dimensionless energy magnitude factor at the right boundary |
| $k$ | thermal conductivity (W m$^{-1}$ K$^{-1}$) |
| $k_0$ | reference thermal conductivity (W m$^{-1}$ K$^{-1}$) |
| $L$ | characteristic length (m) |
| $q$ | heat flux (W m$^{-2}$) |
| $\tilde{q}$ | dimensionless heat flux |
| $T$ | temperature (K) |
| $T_0$ | reference temperature (K) |
| $\tilde{T}$ | dimensionless temperature |
| $t$ | time (s) |
| $t_p$ | dimensionless energy pulse time |
| $Ve$ | Vernotte number |
| $x$ | space direction (m) |
| $\tilde{x}$ | dimensionless space direction |

*Greek symbols*

| | |
|---|---|
| $\alpha_0$ | reference thermal diffusivity (m$^2$ s) |
| $\beta$ | the ratio of relaxation times |
| $\gamma$ | dimensionless coefficient for taking into account of temperature-dependent conductivity |
| $\mu$ | dimensionless absorption coefficient |
| $\rho$ | density (kg m$^{-3}$) |
| $\tau_q$ | heat flux relaxation time (s) |
| $\tau_T$ | temperature relaxation time (s) |

ture gradient, a certain duration of time (heat flux relaxation time, $\tau_q$) takes before the heat flux is established.

The development of non-Fourier models did not stop at the C–V model. In 1995, Tzou [7] introduced another macroscopic model known as the Dual Phase Lagging (DPL) model. The reason for the name was that it was suggested that a temperature gradient relaxation time ($\tau_T$) also exists in addition to heat flux relaxation time ($\tau_q$). This means that when a heat flux is established, duration of time is needed before it can create a temperature gradient. Some empirical studies have confirmed the validity of the model [8,9]. A number of well-established review studies have also addressed the development of non-Fourier models [10,11].

In most studies that have employed non-Fourier models for analyzing the conduction heat transfer, the assumption of constant thermal properties has yielded linear equations, and solving nonlinear equations has been less studied in the field's literature. Nonlinear studies have utilized numerical methods, while challenges such as convergence and computational cost make the use of numerical methods difficult. New techniques have been recently proposed to solve nonlinear problems. They are known as semi-analytical or approximate-analytical methods. Some of the most common analytical-approximate methods include the following: Adomian Decomposition Method (ADM) [12], Homotopy Perturbation Method (HPM) [13,14], Homotopy Analysis Method (HAM) [15], Differential Transform Method (DTM) [16], and Variational Iteration Method (VIM) [17].

Semi-analytical methods have been widely used in solving various heat transfer problems in the past years. Ganji and Rajabi [18] used HPM to solve an unsteady nonlinear convective–radiative equation. Ganji [19] applied HPM to nonlinear equations arising in heat transfer and compared it with the perturbation and numerical methods. Ganji and Sadighi [20] investigated nonlinear heat transfer and porous media equations by HPM and VIM. Chakraverty and Behera [21] investigated the numerical solution of a fractionally damped dynamic system by HPM. HPM was used to present a mathematical model for biofilm inhibition for steady-state conditions by Meena et al. [22]. Gupta et al. [23] studied about the solution of various linear and nonlinear convection–diffusion problems arising in physical phenomena by HPM. Nadeem et al. [24] studied about the effects of nanoparticles on the peristaltic flow of tangent hyperbolic fluid in an annulus which were solved by ADM.

The application of semi-analytical methods to solve the nonlinear problems of non-Fourier heat conduction problems has been reported just in few studies. Torabi et al. [25] applied the homotopy perturbation method (HPM) to solve a nonlinear convective–radiative non-Fourier conduction heat transfer equation with variable specific heat coefficient. Saedodin et al. [26] used the variational iteration method (VIM) to solve the same problem. Differential transformation method (DTM) was applied for analysis of nonlinear convective–radiative hyperbolic lumped systems with simultaneous variation of temperature-dependent specific heat and surface emissivity by Torabi et al. [27]. In all of three mentioned references, the governing equations have been only dependent of time and in fact, ordinary differential equations (ODE) have been solved by semi-analytical methods and to the best knowledge of the authors, nonlinear partial differential equation (PDE) of non-Fourier heat conduction equation has not been solved yet by semi-analytical methods.

In the present paper, the heat transfer phenomenon within a one-dimensional slab subjected to an internal heat source was investigated. The DPL non-Fourier heat conduction model was employed for thermal analysis. A nonlinear equation was obtained since the thermal conductivity was assumed temperature-dependent. The obtained equations were solved by ADM. The main advantage of the ADM is the fact that it provides its user with an analytical approximation, in many cases an exact solution, in a rapidly convergent sequence with elegantly computed terms. Moreover, ADM approximates the nonlinear terms without linearization, perturbation, closure approximations, or discretization methods which can result in massive numerical computations. The application of a