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ORIGINAL ARTICLE

Implementation of fractional order integrator/differentiator on field programmable gate array

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Abstract Concept of fractional order calculus is as old as the regular calculus. With the advent of high speed and cost effective computing power, now it is possible to model the real world control and signal processing problems using fractional order calculus. For the past two decades, applications of fractional order calculus, in system modeling, control and signal processing, have grown rapidly. This paper presents a systematic procedure for hardware implementation of the basic operators of fractional calculus i.e. fractional integrator and derivative, using Grünwald–Letnikov definition, on field programmable gate array (FPGA) in LabVIEW environment. The simulation and hardware implementation results for fractional order integrator and derivative of sinusoid and square waveform signals for some selected fractional orders have been presented. A close agreement between the simulated and the experimental results demonstrated the suitability of FPGA device in fractional order control and signal processing applications. LabVIEW being one of the finest tools for measurement and control, and signal processing applications the fractional order operator implementation is expected to further enhance the capability of the tool to cater to the needs of advanced experimental research employing fractional order operators.

1. Introduction

In recent years, fixed-point operations have been used for hardware implementations to save costs at the enhanced computational speed. Field programmable gate array (FPGA) is one such device. Because of their enormous advantages, there is an increasing trend in the use of FPGA devices as real time hardware targets in industry. FPGA finds potential applications in various domains such as real time measurement and control, signal processing and digital communication. These

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are reconfigurable real time devices available at relatively low costs. On a FPGA device, all of the logic gets implemented in digital hardware alone thus yielding very high processing speed. Modern day FPGAs have clock rates in the range of several MHz and also allow the user to set precise bit settings for accurate computations in addition to the effective memory usage. Additionally, each independent task can utilize a different set of logic on the same FPGA hardware allowing multiple tasks to run simultaneously in contrast to microprocessors. Overall, FPGAs are perfectly suitable for applications in time-critical systems. On the other hand, the disadvantage of FPGAs is that the hardware resources are limited and design requires lots of careful considerations of the precisions required, with specific planning for each individual application. An excellent review of FPGA technologies and their contribution to industrial control applications has been presented in [1]. Various potential application areas were addressed which can exploit the advantages of FPGAs. Benefits of using FPGAs in the case of control applications are then exhibited and supported with case studies of artificial neural networks based control system designs targeting FPGAs.

Fractional calculus has been a topic of theoretical research for scientists and engineers for a very long time. In the past two decades, several potential applications of fractional order calculus have been developed. One of the areas where fractional calculus has been found to be very useful is system modeling. As real objects are generally fractional, system modeling using fractional calculus is much more accurate as compared to the integer order modeling. In a recent work on fractional order modeling, a fractional derivative model was selected to describe the arterial wall mechanics in vivo. The fractional derivative model proved to naturally mimic the elastic modulus spectrum with only four parameters and a reasonable small computational effort [2]. In another work, a nonlinear fractional order model for steer by wire system was presented. Validation of the proposed approach was done in simulation. As reported, this method is very useful in design of a robust controller for steer by wire systems [3]. Fractional order model of Permanent Magnet Synchronous Motor (PMSM) has been proposed in [4]. Simulation and experimental comparisons between the fractional order and the integer order model of PMSM were presented to show the existence of fractional order model on the PMSM speed servo system.

Process control is another area where fractional order control is being sought as an improvement over conventional control. In the conventional PID controller, integral term eliminates the steady-state error but decreases the relative stability of the system and makes it sluggish as well. Derivative term increases the relative stability and makes the system much faster while compromising the sensitivity to noise. Fractional order control, which involves the use of fractional integrals and derivatives instead of classical integer order integral and derivative terms, is able to achieve satisfactory compromises between the above stated positive and negative effects of conventional PID control. Thus, instead of pure integral $\left(\frac{1}{s}\right)$ or pure derivative (s) term, fractional integral/derivative term was used i.e., s^{γ} ($\gamma \in R+$). [5]. In [6], a fractional order PID controller was investigated in simulation for a position servomechanism control system considering actuator saturation and the shaft tensional flexibility. This work claimed that if fractional order PID controller is properly designed and implemented, it will outperform the conventional integer order PID controller. Another such work presents a strategy to tune a fractional order integral and derivative controller satisfying gain and phase margins. This work aimed to apply the tuning procedure proposed to temperature control at selected points in M/S Quanser's heat flow experimental platform. The effectiveness and validity of the technique was experimentally illustrated by comparison with the traditional PI/PID controller based on Ziegler Nichol's tuning method [7]. In [8], explicit analytical expressions for step and impulse responses of a linear fractional-order system with fractional-order controller for open and closed loop were presented. Superiority of the fractional order control over the convention one was demonstrated with the help of an example. A fractional order controller was proposed for a class of fractional order system and a tuning procedure was developed [9]. In the same direction, an intelligent robust fractional surface sliding mode control is proposed for a nonlinear system [10]. A recent research on intelligent fractional order control proposed a novel fractional order fuzzy PID controller. Closed loop performances and controller efforts in the presented cases were compared with conventional PID, fuzzy PID and $PI^{\lambda}D^{\mu}$ controller subjected to different performance indices in simulation. As reported, the fractional order fuzzy PID controller outperformed the others in most cases [11]. Several other recent interesting works on the applications of fractional order control are automatic voltage regulator [12], oscillatory fractional order processes with dead time [13], robotic manipulator [14], binary distillation column [15] and hybrid electric vehicle [16].

Fractional order signal processing and digital filters are also promising application areas of fractional order phenomena. In an early stage work, the behavior of passive *RC* low pass filters when the capacitive element acquires a fractional order was numerically investigated. The effect of the fractional capacitor on time and frequency-domain responses was numerically studied. The research claimed that speed of the response increases with fractional order and that by allowing the capacitor to acquire a fractional order greater than unity; one can achieve the advantages of fast response and linear phase shift over few decades of frequency [17]. Another research presents a general procedure to obtain Butterworth filter of required specifications in the fractional-order domain. The necessary and sufficient conditions for achieving fractional-order Butterworth filter with a specific cutoff frequency were derived as a function of the orders in addition to the transfer function parameters. The effect of equal-orders on the filter bandwidth was discussed showing how the integer-order case is considered as a special case from the proposed procedure. Several passive and active filters were studied to validate the concept such as Kerwin–Huelsman–Newcomb and Sallen–Key filters through numerical simulations. These circuits were tested experimentally using discrete components to model the fractional order capacitor showing good match with the numerical simulations [18].

The mathematics involved in fractional integrator/differentiator is much more complex as compared to integer order integrator/differentiator. Therefore, hardware implementation of fractional operators is also relatively complex. Some techniques for hardware implementation of fractional operators have been proposed and reported in the literature. Several hardware platforms have been used in various applications.