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ORIGINAL ARTICLE

Similarity and analytical solutions of free convective flow of dilatant nanofluid in a Darcian porous medium with multiple convective boundary conditions

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Abstract This paper deals with an analytical solution of free convective flow of dilatant nanofluid past a vertical cone/plate. A two-phase mixture model is used for nanofluid in which the Brownian motion and thermophoretic diffusivities are the important slip mechanisms between solid and liquid phases. The governing transport equations along with physically realistic thermal and mass convective boundary conditions are reduced to similarity equations using relevant similarity transformations before being solved by homotopy analysis method. The effects of the governing parameters (Brownian motion, thermophoresis, convection–conduction, convection–diffusion, Lewis number, buoyancy ratio, and power-law) on the dimensionless velocity, temperature and nanoparticle volume fraction, friction and heat transfer rates are plotted and discussed. It is found that friction factor decreases with the increase in *Le* and *Nr* for both vertical plate and cone. The local Nusselt number decreases with the increase in the thermophoresis and Brownian motion parameters for both the plate and cone. The local Sherwood number increases with the Brownian motion parameter and decreases for thermophoresis parameter. The results have been compared with the published ones and an excellent agreement has been noticed.

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Nomenclature

$\bar{C}$	nanoparticle volume fraction (–)
c_i	arbitrary constant (–)
D_B	Brownian diffusion coefficient ($m^2 s^{-1}$)
D_T	thermophoretic diffusion coefficient ($m^2 s^{-1}$)
$f(\eta)$	dimensionless stream function (–)
g	acceleration due to gravity ($m s^{-2}$)
k	thermal conductivity ($m^2 s^{-1}$)
K	consistency coefficient ($kg m^{-1} s^{-1}$)
K_p	permeability of the porous medium (m^2)
n	power-law index (–)
Nb	Brownian motion parameter (–)
Nc	convection–conduction parameter (–)
Nd	convection–diffusion parameter (–)
Nr	buoyancy-ratio parameter (–)
Nt	thermophoresis parameter (–)
$Nu_{\bar{x}}$	local Nusselt number (–)
$\bar{q}_m$	wall mass flux ($kg m^{-2} s^{-1}$)
$\bar{q}_w$	wall heat flux ($J m^{-2} s^{-1}$)
$Ra_{\bar{x}}$	local Rayleigh number (–)
Sc	Schmidt number (–)
$Sh_{\bar{x}}$	local Sherwood number (–)
$\bar{T}$	temperature (K)
$\bar{u}$, $\bar{v}$	velocity components along and perpendicular to the surface ($m s^{-1}$)
$\bar{x}$, $\bar{y}$	distance along and perpendicular to the surface (m)

Greek letters

α	thermal diffusivity of the porous medium ($m^2 s^{-1}$)
γ	half angle of the cone (radian)
$\hbar$	auxiliary nonzero parameter (–)
$\mathcal{L}$	auxiliary linear operator (–)
$\mathcal{N}$	nonlinear operator (–)
η	similarity variable (–)
θ	dimensionless fluid temperature (–)
φ	rescaled nanoparticle volume fraction (–)
ρ	density ($kg m^{-3}$)
(ρc)	heat capacity ($J kg^{-3} K^{-1}$)
τ	the ratio of the effective heat capacity of the nanoparticle material and heat capacity of the fluid (–)
υ	fluid kinematic viscosity ($m^2 s^{-1}$)
ψ	stream function (–)

Subscripts

f	fluid
p	nanoparticle
w	wall condition
∞	infinity condition

Superscript

$'$	differentiation with respect to η

1. Introduction

Including a dispersion of nanoparticles, nanofluid is a liquid in which the nanoparticles have been suspended in it without settlement and this can be pointed as the difference between nanoparticles and conventional particles [1]. Some researchers considered the free convective boundary-layer flow of nanofluids subject to different boundary conditions. Chamkha et al. [2] studied free convection past an isothermal sphere in a Darcy porous medium with a nanofluid and then presented numerical results for mass transfer rate, friction factor and surface heat transfer rate. Considering the effects of Brownian motion and thermophoresis, Nield and Kuznetsov [3,4] investigated the Cheng–Minkowycz problem and thermal instability in a porous medium (Darcy and Brinkman models) analytically. Xuan and Li [5] studied the behavior of nanofluid in turbulent flow through the tubes experimentally. According to their published results, the Reynolds number and volume fraction of nanoparticles can affect the convective heat transfer coefficient and Nusselt number of nanofluids. Hady et al. [6] studied the effect of radiation parameter over a nonlinear stretching sheet in a viscous flow of a nanofluid. An implicit finite-difference method was employed by Khan and Pop to study the steady nanofluid flow past a stretching surface [7]. Beg et al. [8] presented a comparative numerical solution for single-phase and two-phase models for Bio-nanofluid transport phenomena. Noghrehabadi et al. [9] considered the partial slip condition of nanofluids past a stretching sheet with the constant wall temperature. Bachok et al. [10] presented the results for a uniform free stream of a steady nanofluid flow over a semi-infinite flat plate. Abu-Nada et al. [11] took into consideration the effects of variable properties in a natural convective nanofluid flow. Rashidi and Erfani [12] used modified differential transform method to study the nano boundary-layer flow over the stretching surfaces with Navier boundary condition. Nadeem and Lee [13] studied the steady flow of a nanofluid over an exponential stretching surface. Stagnation-point nanofluid flow over a surface was studied by Rashidi et al. [14] via DTM-Padé. Khan et al. [15] considered the effect of momentum slip on Double-Diffusive natural convection of a nanofluid over a vertical plate.

Melts of polymers, biological solutions and paint which are non-Newtonian fluids play an important role in many industrial applications such as alternative energy technologies, microfluidic devices and biomedical devices [16]. The boundary-layer flow of non-Newtonian power-law nanofluids past a linearly stretching sheet was studied by Uddin et al. [17] with a linear hydrodynamic slip boundary condition numerically using Runge–Kutta–Fehlberg fourth-fifth order. Sheu [18] investigated the thermal instability in a porous medium horizontal layer saturated with a viscoelastic nanofluid by employing Oldroyd-B viscoelastic model. Akbar and Nadeem [19] used Eyring–Prandtl fluid model to study flow of a nanofluid in a diverging tube. To study heat transfer of Casson non-Newtonian fluid flow past a horizontal circular cylinder, Prasad et al. [20] employed the Keller box finite difference