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ORIGINAL ARTICLE

Three dimensional boundary layer flow of a viscoelastic nanofluid with Soret and Dufour effects

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Abstract The present research focuses on the three-dimensional flow of viscoelastic fluid in the presence of Soret and Dufour effects. Effects of thermophoresis and Brownian motion are taken into account. Appropriate similarity transformations lead to nonlinear ordinary differential equations. Solution expressions of velocity, temperature and nanoparticle concentration are computed via homotopy analysis method (HAM). Convergence of obtained solutions is analyzed graphically and numerically. Results are plotted and analyzed for the dimensionless velocities, temperature and nanoparticle concentration. Values of local Nusselt and Sherwood numbers are examined through tabular form. It is observed that Temperature field is enhanced for the larger Brownian motion parameter and an increase in Dufour number gives rise to the temperature and thermal boundary layer thickness.

1. Introduction

Heat transfer mechanism has an important role in many engineering and industrial fields because cooling and heating processes are involved in such fields. An increase in heat transfer rate is quite essential. It reduces the process time of work and length of the work life of equipment. Various methods are proposed in the past to increase the heat transfer efficiency rate. Some methods involve extended surfaces, applications of vibration to the heat transfer surfaces and usage of micro-channels is studied by Kwak and Kim [1]. There is another method to increase the heat transfer efficiency by increasing the thermal conductivity of the working fluids as mentioned by Ramzan [2]. Most commonly used working fluids such as water, engine oil, and ethylene glycol have lower thermal conductivity compared to the thermal conductivity of solids. Solids of higher thermal conductivity can be utilized to increase the thermal conductivity of the base fluid by emerging small solid particles in the fluid. Emerging of such particles in the base fluid is known as nanofluid. Such fluids have several applications in biomedical and engineering applications in cooling, cancer therapy and process industries. A tremendous work on nanofluids can be seen in Refs. [3–12].

The boundary layer flow over a continuously stretching surface is commonly encountered in various industrial and

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engineering processes such as materials manufactured by extrusion of plastic sheets and materials traveling between a windup roll and feed roll. Numerous studies have been done on the two-dimensional boundary layer flow over a stretching surface. Not much attention is paid to three-dimensional boundary layer flow induced by stretching surface. Wang [13] was the first who investigated the three-dimensional boundary layer flow of viscous fluid over a bidirectional stretching surface and provided exact solution. Later on Ariel [14] constructed the homotopy perturbation solution of Wang [13] flow problem. Hayat et al. [15] computed the series solutions of three-dimensional of an Oldroyd-B fluid in the presence of convective boundary conditions. Hydromagnetic flow of Maxwell fluid induced by a bidirectional stretching surface with variable thermal conditions is investigated by Shehzad et al. [16]. Ahmad et al. [17] studied the hydromagnetic flow of three-dimensional viscous fluid in the presence of heat generation/absorption. Some useful studies may be found through [18–20].

In all the abovementioned studies, the influences of Soret and Dufour effects are not addressed because they have smaller order of magnitude than the effects described by Fourier's and Fick's laws. Eckert and Drake [21] investigated that there are several cases where such effects cannot be neglected. Soret and Dufour effects are important for the fluids which have light molecular weight or medium molecular weight. Turkyilmazoglu and Pop [22] discussed the Soret and heat source effects on unsteady radiative flow of viscous fluid induced by an impulsively started infinite vertical plate. Combined effects of partial slip, thermal diffusion and diffusion thermo, in steady flow of viscous fluid generated by a rotating disk were examined by Rashidi et al. [23]. Alsaedi et al. [24] analyzed the Soret and Dufour effects on two-dimensional boundary layer flow of second grade fluid over a stretching surface. Stagnation point flow of Jeffery fluid induced by a convectively heated sheet under Soret and Dufour effects is addressed by Shehzad et al. [25].

The aim of the present work is to study the effects of Soret and Dufour on the boundary layer flow of viscoelastic fluid in the presence of nanoparticles and chemical reaction. No such analysis is provided in the literature. We used homotopy analysis method (HAM) [26–33] to discuss the solution of velocities, temperature and nanoparticle concentration. The heat transfer rate and nanoparticles concentration transfer rate at the walls are computed numerically.

2. Mathematical formulation

We consider the three-dimensional flow of an incompressible viscoelastic nanofluid over a stretching surface at $z = 0$. The motion in fluid is induced due to the stretching of surface. The heat and nanoparticle mass transfer characteristics have been considered when both Soret and Dufour effects are present. The continuity, momentum, energy and nanoparticle concentration equations for the boundary layer flow are given below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu\frac{\partial^2 u}{\partial z^2}-K\left(u\frac{\partial^3 u}{\partial x\partial z^2}+w\frac{\partial^3 u}{\partial z^3}-\begin{pmatrix}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial z^2}+\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2}\\ 2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x\partial z}+2\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2}\end{pmatrix}\right), \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\nu\frac{\partial^2 v}{\partial z^2}-K\left(u\frac{\partial^3 v}{\partial x\partial z^2}+w\frac{\partial^3 v}{\partial z^3}-\begin{pmatrix}\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial z^2}+\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial z^2}\\ 2\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial x\partial z}+2\frac{\partial w}{\partial z}\frac{\partial^2 v}{\partial z^2}\end{pmatrix}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha_m\frac{\partial^2 T}{\partial z^2}+\frac{D_B k_T}{C_s C_\rho}\frac{\partial^2 C}{\partial z^2}+\tau\left(D_B\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2\right), \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D_B\frac{\partial^2 C}{\partial z^2}-K_1(C-C_\infty)+\frac{Dk_T}{T_m}\frac{\partial^2 T}{\partial z^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial z^2}, \tag{5}$$

where u, v and w denote the velocity components in the x, y and $z$directions, respectively, $\nu = \frac{\mu}{\rho}$ the kinematic viscosity, K the material fluid parameter, C the concentration of the nanoparticles species, D_B the Brownian diffusion coefficients, K_1 the reaction rate, k_T the thermal diffusion, T the temperature, C_ρ the specific heat, C_s the concentration susceptibility, α_m the thermal diffusivity, and T_m the fluid mean temperature.

The boundary conditions for the present flow analysis can be written as

$$u = u_w(x) = ax, \quad v = v_w(y) = by, \quad w = 0, \quad T = T_w, \quad C = C_w \quad \text{at } z = 0, \tag{6}$$

$$u \to 0, \quad v \to 0, \quad \frac{\partial u}{\partial z} \to 0, \quad \frac{\partial v}{\partial z} \to y, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{at } z \to \infty, \tag{7}$$

where C_w denotes the nanoparticles concentration at the surface, C_∞ is the nanoparticle concentration far away from the sheet, T_w is the surface temperature and T_∞ is the temperature far away from the surface. Using

$$\eta=\sqrt{\frac{a}{\nu}}z, \quad u=axf'(\eta), \quad v=ayg'(\eta), \quad w=-\sqrt{a\nu}\{f(\eta)+g(\eta)\}, \quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}, \quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}. \tag{8}$$

Eq. (1) is identically satisfied while Eqs. (2)–(7) are reduced to

$$f''' - (f')^2 + (f+g)f'' + K_0\left((f+g)f'''' + (f''-g'')f'' - 2(f'+g')f'''\right) = 0, \tag{9}$$

$$g''' - (g')^2 + (f+g)g'' + K_0\left((f+g)g'''' + (f''-g'')g'' - 2(f'+g')g'''\right) = 0, \tag{10}$$

$$\theta'' + \Pr((f+g)\theta' + (N_t\theta' + N_b\phi')\theta' + Du\phi'') = 0, \tag{11}$$

$$\phi'' + \Pr Le((f+g)\phi' - \gamma\phi + Sr\theta'') + \left(\frac{N_t}{N_b}\right)\theta'' = 0, \tag{12}$$

$$f(0) = 0, \quad g(0) = 0, \quad f'(0) = 1, \quad g'(0) = c, \quad \theta(0) = 1, \quad \phi(0) = 1, \quad f'(\infty) \to 0, \quad f''(\infty) \to 0, \quad g'(\infty) \to 0, \quad g''(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0. \tag{13}$$