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Bending analysis of embedded nanoplates based on the integral formulation of Eringen's nonlocal theory using the finite element method

R. Ansari *, J. Torabi **, A. Norouzzadeh

Department of Mechanical Engineering, University of Guilan, P.O. Box 3756, Rasht, Iran

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ABSTRACT

Due to the capability of Eringen's nonlocal elasticity theory to capture the small length scale effect, it is widely used to study the mechanical behaviors of nanostructures. Previous studies have indicated that in some cases, the differential form of this theory cannot correctly predict the behavior of structure, and the integral form should be employed to avoid obtaining inconsistent results. The present study deals with the bending analysis of nanoplates resting on elastic foundation based on the integral formulation of Eringen's nonlocal theory. Since the formulation is presented in a general form, arbitrary kernel functions can be used. The first order shear deformation plate theory is considered to model the nanoplates, and the governing equations for both integral and differential forms are presented. Finally, the finite element method is applied to solve the problem. Selected results are given to investigate the effects of elastic foundation and to compare the predictions of integral nonlocal model with those of its differential nonlocal and local counterparts. It is found that by the use of proposed integral formulation of Eringen's nonlocal model, the paradox observed for the cantilever nanoplate is resolved.

1. Introduction

The nonlocal elasticity theory is extensively used to analyze the mechanical characteristics of nanostructures such as nanobeams, nanoplates and nanotubes. The mechanical behavior of structures at nanoscale is size-dependent [1], and the classical continuum mechanics cannot capture the size effects. Therefore, different modified continuum mechanics theories are employed to investigate the structural characteristics of structures at micro and nano scales [2–6]. Among the non-classical continuum mechanics theories, the nonlocal elasticity theory firstly proposed by Eringen [7] and Eringen and Edelen [8] has been employed by many researchers to study the nanostructures. According to nonlocal theory, the stress at the reference point is a function of strain field at all points of the domain. For more information about the development of nonlocal elasticity theory, one can refer to [9–13].

The first version of nonlocal model presented by Eringen [7] was in the integral form and the nonlocal influences could be taken into account using arbitrary kernel functions. Since managing of the associated integro-partial differential equations was mathematically difficult, the differential form of nonlocal elasticity theory was proposed [10] for a specific kernel function (Green function of linear differential operator). Due to its easier mathematical treatment in comparison to the integral form, the differential form of nonlocal elasticity theory is widely employed to study the mechanics of nanostructures. The work presented by Peddieson et al. [14], which shows the application of differential version of Eringen's nonlocal theory in nanotechnology, attracted a great deal of attention among the scholars to employed this model in their studies on the nanobeams [15–21], nanoplates [22–33], nanotubes [34–43] and nanocones [44–46]. For instance, Demir and Civalek [20] presented the vibration analysis of embedded nanobeams based on a new nonlocal finite element formulation using the Hermitian cubic shape functions. The new analytical approach according to Hamiltonian-based model and the nonlocal theory was reported by Rong et al. [31] for free and forced vibration and buckling of nanoplates. Also, Arani and Jalaei [33] studied the influences of longitudinal magnetic field on the dynamic response of viscoelastic graphene sheet based on the nonlocal model and sinusoidal shear deformation theory. In addition, comprehensive review study on the application of the nonlocal elasticity models in modeling of the carbon nanotubes and graphenes are presented in Ref. [47].

However, in some cases it was seen that the differential form of Eringen's nonlocal theory presents the paradoxical results. For instance, it was reported that on the basis of the differential nonlocal model, as the nonlocal parameter increase, the cantilever nanobeams under the distributed loading conditions show the stiffening effect [48].

* Corresponding author.
** Corresponding author.
E-mail addresses: r_ansari@guilan.ac.ir (R. Ansari), Torabi@webmail.guilan.ac.ir (J. Torabi).

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Accordingly, various studies have been performed to solve Eringen's integral nonlocal model or propose a modified nonlocal models in order to resolve the reported paradoxes. Employing an integral-based Eringen's nonlocal model for the cantilever nanobeam under point load, leads to the response identical to the classical cantilever beam model without any small scale effect. To overcome this paradox, a gradient elastic model and an integral nonlocal elastic model were presented by Challamel and Wang [49] based on combining the local and the nonlocal curvatures in the constitutive elastic relation. Furthermore, Challamel et al. [50] indicated that considering nonself-adjointness of Eringen's differential model leads to stiffening effect on the vibration analysis of the clamped-free beams with the increase of small length scale coefficient. Moreover, an anlytical approach was rcently presented by Fernández-Sáez et al. [51] to solve the integral form of Eringen's nonlocal elasticity theory for the bending of Euler-Bernoulli nanobeams. The results revealed that with integral formulation, the paradox that appears when solving the cantilever beam with the differential form of the Eringen model is solved. Employing the finite element method, Tuna and Kirca [52] studied the bending, free vibration and buckling of nanobeams based on the integral form of Eringen's nolocal theory along with Euler-Bernoulli beam model.

In the present paper, the bending of embedded nanoplates subjected to static uniform loading is investigated within the framework of Eringen's integral nonlocal model. The Winkler- and Pasternak-type elastic foundations are taken into account. On the basis of the first order shear deformation plate theory, the governing equations for both integral and differential forms are derived. The finite element method is also used to solve the problem and to study the bending behavior of embedded nanoplates under various boundary conditions. Comparison studies are also conducted between the results of integral form of Eringen's nonlocal theory and those of nonlocal differential and local models.

2. Integral model of nonlocal elasticity

2.1. Derivation of formulation

According to Eringen's nonlocal elasticity theory, the general form of constitutive equations of plates are given as follows [7].

$$t_{ij}(\overline{\mathbf{x}},z)=\lambda\delta_{ij}\varepsilon_{kk}(\overline{\mathbf{x}},z)+2\mu\varepsilon_{ij}(\overline{\mathbf{x}},z)=C_{ijkl}\varepsilon_{kl}(\overline{\mathbf{x}},z) \tag{1}$$

$$\sigma_{ij}(\mathbf{x},z)=\int_A k(|\mathbf{x}-\overline{\mathbf{x}}|,\kappa)\,t_{ij}(\overline{\mathbf{x}},z)\,dA(\overline{\mathbf{x}}) \tag{2}$$

where ${}_a t_{ij}$ and σ_{ij} are the components of local and nonlocal stress tensors respectively, ε_{ij} is the strain tensor component, C_{ijkl} is fourth order elasticity tensor and δ_{ij} stands for Kronecker delta. In addition, $\overline{\mathbf{x}}=(\overline{x},\overline{y})$ is all points on domain, $\mathbf{x}=(x,y)$ denotes reference point, k presents the kernel function, $\kappa=e_0a$ is length parameter and $|\mathbf{x}-\overline{\mathbf{x}}|$ shows the neighborhood distance.

Considering above, the elastic strain energy of nanoplate and its variation can be presented as

$$\begin{aligned}\Pi_s&=\frac{1}{2}\int_z\int_A\sigma_{ij}(\mathbf{x},z)\varepsilon_{ij}(\mathbf{x},z)\,dA(\mathbf{x})dz\\&=\frac{1}{2}\int_z\int_A\left(\int_{\overline{A}}k(|\mathbf{x}-\overline{\mathbf{x}}|,\kappa)\,t_{ij}(\overline{\mathbf{x}},z)\,dA(\overline{\mathbf{x}})\right)\varepsilon_{ij}(\mathbf{x},z)\,dA(\mathbf{x})dz\end{aligned} \tag{3}$$

$$\begin{aligned}\delta\Pi_s&=\int_z\int_A\sigma_{ij}(\mathbf{x},z)\delta\varepsilon_{ij}(\mathbf{x},z)\,dA(\mathbf{x})dz\\&=\int_z\int_A\left(\int_{\overline{A}}k(|\mathbf{x}-\overline{\mathbf{x}}|,\kappa)\,t_{ij}(\overline{\mathbf{x}},z)\,dA(\overline{\mathbf{x}})\right)\delta\varepsilon_{ij}(\mathbf{x},z)\,dA(\mathbf{x})dz\end{aligned} \tag{4}$$

Considering the first order shear deformation theory, the displacement field of plate is introduced as

$$\begin{aligned}u_1(\overline{x},\overline{y},z,t)&=u(\overline{x},\overline{y},t)+z\psi(\overline{x},\overline{y},t),\ u_2(\overline{x},\overline{y},z,t)\\&=v(\overline{x},\overline{y},t)+z\phi(\overline{x},\overline{y},t),\ u_3(\overline{x},\overline{y},z,t)=w(\overline{x},\overline{y},t)\end{aligned} \tag{5}$$

which can be rewritten as follows

$$\begin{aligned}&\mathbf{u}=\mathbf{p}(z)\overline{\mathbf{q}}, \qquad \mathbf{u}(\overline{\mathbf{x}},z)=[u_1\ \ u_2\ \ u_3]^{\mathrm{T}}\\&\mathbf{p}(z)=\begin{bmatrix}1&0&0&z&0\\0&1&0&0&z\\0&0&1&0&0\end{bmatrix},\ \ \overline{\mathbf{q}}=[u(\overline{\mathbf{x}},t)\ \ v(\overline{\mathbf{x}},t)\ \ w(\overline{\mathbf{x}},t)\ \ \psi(\overline{\mathbf{x}},t)\ \ \phi(\overline{\mathbf{x}},t)]^{\mathrm{T}}\end{aligned} \tag{6}$$

The constitutive relation presented in Eq. (2) can be given in matrix-vector form as

$$\boldsymbol{\sigma}(\mathbf{x},z)=\int_{\overline{A}}k(|\mathbf{x}-\overline{\mathbf{x}}|,\kappa)\mathbf{C}\overline{\boldsymbol{\varepsilon}}(\overline{\mathbf{x}},z)\,dA(\overline{\mathbf{x}}) \tag{7}$$

in which

$$\boldsymbol{\sigma}(\mathbf{x},z)=\begin{bmatrix}\sigma_{xx}(\mathbf{x},z)\\\sigma_{yy}(\mathbf{x},z)\\\sigma_{xy}(\mathbf{x},z)\\\sigma_{yz}(\mathbf{x},z)\\\sigma_{xz}(\mathbf{x},z)\end{bmatrix},\ \mathbf{C}=\begin{bmatrix}\lambda+2\mu&\lambda&0&0&0\\&\lambda+2\mu&0&0&0\\&&\mu&0&0\\&&&k_s\mu&0\\sym.&&&&k_s\mu\end{bmatrix},\ \overline{\boldsymbol{\varepsilon}}(\overline{\mathbf{x}},z)=\begin{bmatrix}\varepsilon_{xx}(\overline{\mathbf{x}},z)\\\varepsilon_{yy}(\overline{\mathbf{x}},z)\\2\varepsilon_{xy}(\overline{\mathbf{x}},z)\\2\varepsilon_{yz}(\overline{\mathbf{x}},z)\\2\varepsilon_{xz}(\overline{\mathbf{x}},z)\end{bmatrix} \tag{8}$$

where $\boldsymbol{\sigma}$, $\overline{\boldsymbol{\varepsilon}}$ and $\mathbf{C}$ are the stress vector, strain vector and material stiffness matrix, respectively.

According to the displacement field, the strain-displacement relationships is presented as

$$\begin{aligned}&\varepsilon_{xx}(\overline{\mathbf{x}},z)=\frac{\partial u}{\partial\overline{x}}+z\frac{\partial\psi}{\partial\overline{x}},\ \varepsilon_{yy}(\overline{\mathbf{x}},z)=\frac{\partial v}{\partial\overline{y}}+z\frac{\partial\phi}{\partial\overline{y}},\\&2\varepsilon_{xy}(\overline{\mathbf{x}},z)=\frac{\partial u}{\partial\overline{x}}+\frac{\partial v}{\partial\overline{y}}+z\left(\frac{\partial\psi}{\partial\overline{x}}+\frac{\partial\phi}{\partial\overline{y}}\right),\\&2\varepsilon_{yz}(\overline{\mathbf{x}},z)=\left(\frac{\partial w}{\partial\overline{y}}+\phi\right),\ \ 2\varepsilon_{xz}(\overline{\mathbf{x}},z)=\left(\frac{\partial w}{\partial\overline{x}}+\psi\right)\end{aligned} \tag{9}$$

Subsequently, the strain vector is presented as

$$\overline{\boldsymbol{\varepsilon}}(\overline{\mathbf{x}},z)=\mathbf{P}(z)\overline{\mathbf{E}}\overline{\mathbf{q}} \tag{10}$$

in which

$$\mathbf{P}(z)=\begin{bmatrix}1&0&0&z&0&0&0&0\\0&1&0&0&z&0&0&0\\0&0&1&0&0&z&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{bmatrix} \tag{11}$$

$$\overline{\mathbf{E}}=\begin{bmatrix}\partial_{\overline{x}}&0&0&0&0\\0&\partial_{\overline{y}}&0&0&0\\\partial_y&\partial_{\overline{x}}&0&0&0\\0&0&0&\partial_{\overline{x}}&0\\0&0&0&0&\partial_{\overline{y}}\\0&0&0&\partial_{\overline{y}}&\partial_{\overline{x}}\\0&0&\partial_{\overline{y}}&0&1\\0&0&\partial_{\overline{x}}&1&0\end{bmatrix},\ \ \partial_{\overline{x}}=\frac{\partial}{\partial\overline{x}}\ \text{and}\ \ \partial_{\overline{y}}=\frac{\partial}{\partial\overline{y}}, \tag{12}$$

Now, Eqs. (3) and (4) can be represented as

$$\Pi_s=\frac{1}{2}\int_z\int_A\boldsymbol{\varepsilon}^{\mathrm{T}}(\mathbf{x},z)\boldsymbol{\sigma}(\mathbf{x},z)\,dA(\mathbf{x})dz \tag{13}$$

$$\delta\Pi_s=\int_z\int_A\delta\boldsymbol{\varepsilon}^{\mathrm{T}}(\mathbf{x},z)\boldsymbol{\sigma}(\mathbf{x},z)\,dA(\mathbf{x})dz \tag{14}$$