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Comparative investigation of five nanoparticles in flow of viscous fluid with Joule heating and slip due to rotating disk

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ABSTRACT

Present article addresses the comparative study for flow of five water based nanofluids. Flow in presence of Joule heating is generated by rotating disk with variable thickness. Nanofluids are suspension of Silver (Ag), Copper (Cu), Copper oxide (CuO), Aluminum oxide or Alumina (Al_2O_3), Titanium oxide or titania (TiO_2) and water. Boundary layer approximation is applied to partial differential equations. Using Von Karman transformations the partial differential equations are converted to ordinary differential equations. Convergent series solutions are obtained. Graphical results are presented to examine the behaviors of axial, radial and tangential velocities, temperature, skin friction and Nusselt number. It is observed that radial, axial and tangential velocities decay for slip parameters. Axial velocity decays for larger nanoparticle volume fraction. Effect of nanofluids on velocities dominant than base material. Temperature rises for larger Eckert number and temperature of silver water nanofluid is more because of its higher thermal conductivity. Surface drag force reduces for higher slip parameters. Transfer of heat is more for larger disk thickness index.

1. Introduction

Research for steady flow by a rotating disk with heat transfer has become one of the most popular topic in academia due to its extensive technical applications such as computer storage, geothermal industry, rotating machinery, turbo-machinery, lubrication, chemical process, jet motors, food processing, electric power generating system and turbine system. Pioneer work on flow due to rotating disk is done by Karman [1]. He provided transformations which help us to construct ordinary differential equation from Navier Stokes equations. Cochran [2] also used these transformations to examine rotating disk flow by numerical integration method. Rotating flow by two disks is firstly examined by Stewartson [3]. Mellor et al. [5] and Chapple and Stokes [4] studied flow between rotating disks. Arora and Stokes [6] analyzed the heat transfer between two rotating disks. Kumar et al. [7] described flow between porous stationary disk and solid rotating disk. Hayat et al. [8] discovered the effect of thermally stratification on fluid flow between rotating stretchable disks. Radiative flow of carbon nanotubes between rotating stretchable disks with convective conditions has been examined by Imtiaz et al. [9]. Investigation of entropy generation in MHD radiative flow with viscous dissipation and Joule heating is done by Hayat et al. [10].

Surfaces of variable thickness have applications in engineering particularly mechanical, civil, marine, architectural and aeronautical processes. It also helps to reduce the weight of structural elements and improve the utilization of material. It is noted that little information is present for flow due to stretching surfaces with variable thickness. Fluid flow over a variable thicked surface with Cattaneo-Christov heat flux and variable thermal conductivity is presented by Hayat et al. [11]. Stagnation point flow of Casson fluid towards variable thicked surface with radiation is analyzed by Ramesh et al. [12]. Ostwald-de Waele fluid flow and heat transfer by a rotating disk with variable thickness and decreasing index is illustrated by Xun et al. [13]. Effect of stagnation point flow and carbon nanotubes towards a nonlinear stretching sheet with variable thickness is explored by Hayat et al. [14]. Fang et al. [15] worked on flow over a stretching sheet with variable thickness. Hayat et al. [16] studied the effect of melting heat transfer in flow of Williamson nanofluid by nonlinear variable thicked surface. Bending collapse of square tube with variable thickness is illustrated by Zhang et al. [17].

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Nanofluid is mixture of nanometer sized materials such as nanorods, nanotubes and nanoparticles in conventional fluids e.g. ethylene-glycol, oil and water etc. Initially Choi [18] used the word nanofluid. Commonly used nanoparticles are carbon nanotubes, metals (Fe, Au, Ag, Cu), metallic oxides (CuO, Al_2O_3, TiO_2) nitrides (SiN, SiC, TiC) etc. Nanofluids are used in automobiles due to its feature of enhancing heat transfer rate. Nanofluids are also used to cool down the high heat lux devices in welding equipments such as high power laser diode arrays and microwave tubes. For nuclear applications the nanofluids measurement of critical heat flux in a forced convection loop is very valuable. Some more usage of nanofluid are electronic system like microprocessors, MEMS and transportation to energy production. Si et al. [19] described the pseudo-plastic power law nanofluids flow and heat transfer past a stretching plate. Hayat et al. [20] mentioned the effects of mixed convection and nonlinear thermal radiation in flow of silver and copper water nanofluids. Sheikholeslami et al. [21] examined the effect of radiation on nanofluid flow and heat transfer. Flow of nanofluid with inclined magnetic field, nonlinear thermal radiation and heat source/sink is examined by Hayat et al. [22]. Khan et al. [23] worked on convective nanofluid flow due to flat plate in moving free stream. Shahzad et al. [24] described the flow of Oldroyd-B nanofluid over a radiative surface. Effect of Newtonian heating in flow of nanofluid due to permeable cylinder is analyzed by Hayat et al. [25]. Some other recent literature for nanofluid can be seen in Refs. [26–29].

The formation and use of microdevices remain a hot debated and challenging topic of researchers. Microdevices of high efficiency and small size like microvalves, microsenses and micropumps are advantages of using MEMS and NEMS. Occurrence of wall slip in an array of complex fluids like polymer solution, suspension, foam and emulsion cannot be ignored. There are also technological applications of fluid that exhibit the boundary slip like internal cavities and polishing of artificial heart valves. Flow and heat transfer of nanofluids in microchannel in slip and non-slip flow regimes are analyzed by Akbarinia et al. [30]. Mahmoud and Waheed [31] considered the slip and heat generation effects in flow of micropolar fluid over to stretching surface. Nanofluid flow in presence of velocity slip and nonlinear thermal radiation is illustrated by Hayat et al. [32]. Malvandi and Ganji [33] examined the partial slip effect in flow of nanofluid inside a circular microchannel with Brownian motion and thermophoresis. Fang and Aziz [34] discussed the flow over a stretching sheet with second-order slip velocity. Rashidi et al. [35] investigated the slip flow due to rotating porous disk with variable properties and entropy generation.

Much attention in past has been given the flow due to rotating disk with negligible thickness. Here we study the water based nanofluids flow in presence of slip effect and Joule heating. Flow of nanofluids by a disk with variable thickness in presence of slip and Joule heating is not studied yet. Our main objective is to fill this void. MHD effects are also taken in account. Equations are solved by using homotopy analysis method [36–50]. Graphical technique is used to elaborate the impact of involved parameters on the velocity, temperature, skin friction coefficient and Nusselt number. Comparison for five nanoparticles is presented and analyzed.

2. Modeling

We analyze the incompressible water based nanofluids flow by a rotating disk with angular velocity Ω and stretching rate c. Nanofluids are suspension of nanoparticles like silver (Ag), copper (Cu), copper oxide (CuO), aluminum oxide or alumina (Al_2O_3) and titanium oxide or titania (TiO_2) with water as base fluid. The disk of variable thickness is considered at $z = a\left(\frac{r}{R_0}+1\right)^{-\varsigma}$. Disk and ambient temperatures are denoted by $\widehat{T}_w$ and $\widehat{T}_\infty$ respectively. Magnetic field of strength B_0 is applied parallel to $z-$ axis. Effects of Joule heating are also analyzed. Slip flow regime is also considered. We consider cylindrical coordinates $(r,\ \theta,\ z)$ see physical model in Fig. 1. Under the assumptions $\frac{\partial\widehat{p}}{\partial r} = \frac{\partial\widehat{p}}{\partial z} = 0$, $O(\widehat{u}) = O(\widehat{v}) = O(r) = O(1)$ and $O(\widehat{w}) = O(z) = O(\delta)$ the equations for flow and heat transfer are as follows [10,27]:

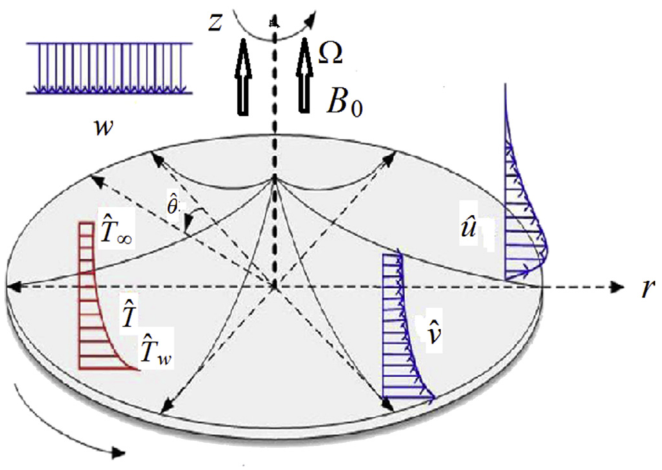

Fig. 1. Geometry of flow analysis.

$$\frac{\partial\widehat{u}}{\partial r} + \frac{\widehat{u}}{r} + \frac{\partial\widehat{w}}{\partial z} = 0, \tag{1}$$

$$\widehat{u}\frac{\partial\widehat{u}}{\partial r} + \widehat{w}\frac{\partial\widehat{u}}{\partial z} - \frac{\widehat{v}^2}{r} = \nu_{nf}\frac{\partial^2\widehat{u}}{\partial z^2} - \frac{\sigma_{nf}}{\rho_{nf}}B_0^2\widehat{u}, \tag{2}$$

$$\widehat{u}\frac{\partial\widehat{v}}{\partial r} + \widehat{w}\frac{\partial\widehat{v}}{\partial z} + \frac{\widehat{u}\widehat{v}}{r} = \nu_{nf}\frac{\partial^2\widehat{v}}{\partial r^2} - \frac{\sigma_{nf}}{\rho_{nf}}B_0^2\widehat{v}, \tag{3}$$

$$(\rho c_p)_{nf}\left(\widehat{u}\frac{\partial\widehat{T}}{\partial r} + \widehat{w}\frac{\partial\widehat{T}}{\partial z}\right) = k_{nf}\frac{\partial^2\widehat{T}}{\partial z^2} + \sigma_{nf}B_0^2(\widehat{u}^2 + \widehat{v}^2), \tag{4}$$

with boundary conditions

$$\widehat{u} = rc + \lambda_1\frac{\partial\widehat{u}}{\partial z},\quad \widehat{v} = r\Omega + \lambda_2\frac{\partial\widehat{v}}{\partial z},\quad \widehat{w} = 0,\quad \widehat{T} = \widehat{T}_w \text{ at } z = a\left(\frac{r}{R_0}+1\right)^{-\varsigma},$$

$$\widehat{u} = 0,\quad \widehat{v} = 0,\quad \widehat{w} = 0,\quad \widehat{T} = \widehat{T}_\infty \text{ when } z \to \infty, \tag{5}$$

where a is the thickness coefficient of the disk which is very small, R_0 the feature radius and ς the disk thickness index, c is the stretching rate and λ_1 and λ_2 are velocity slip coefficients, The effective nanofluid dynamic viscosity μ_{nf}, density ρ_{nf}, heat capacitance $(\rho c_p)_{nf}$, thermal conductivity k_{nf} and electrical conductivity σ_{nf} are defined as

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \tag{6}$$

$$\rho_{nf} = \rho_f(1-\phi) + \rho_s\phi, \tag{7}$$

$$(\rho c_p)_{nf} = (\rho c_p)_f(1-\phi) + (\rho c_p)_s\phi, \tag{8}$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)}, \tag{9}$$

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}, \tag{10}$$

Von Karman transformations are

$$\widehat{u} = r^*R_0\Omega\widetilde{F}(\eta),\quad \widehat{v} = r^*R_0\Omega\widetilde{G}(\eta),\quad \widehat{w} = R_0\Omega(1+r^*)^{-\varsigma}\left(\frac{\Omega R_0^2\rho_f}{\mu_f}\right)^{\frac{-1}{n+1}}\widetilde{H}(\eta)$$