Alexandria Engineering Journal (2015) 54, 889–896

HOSTED BY

ELSEVIER

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

Effect of slip on Herschel–Bulkley fluid flow through narrow tubes

Nallapu Santhosh [a], G. Radhakrishnamacharya [a], Ali J. Chamkha [b,*]

[a] *Department of Mathematics, National Institute of Technology, Warangal, India*
[b] *Mechanical Engineering Department, Prince Mohammad Bin Fahd University, Al-Khobar 31952, Saudi Arabia*

Received 5 June 2015; revised 13 July 2015; accepted 22 July 2015
Available online 10 August 2015

KEYWORDS

Effective viscosity;
Herschel–Bulkley fluid;
Hematocrit;
Plug flow;
Slip

Abstract A two-fluid model of Herschel–Bulkley fluid flow through tubes of small diameters and slip at the wall is studied. It is assumed that the core region consists of Herschel–Bulkley fluid and Newtonian fluid in the peripheral region. Following the analysis of Chaturani and Upadhya, the equations of motion have been linearized and analytical solution for velocity, flow flux, effective viscosity, core hematocrit and mean hematocrit has been obtained. The expressions for all these flow relevant quantities have been numerically computed by using Mathematica software and the effects of various relevant parameters on these flow variables have been studied. It is found that effective viscosity, core hematocrit and mean hematocrit of Newtonian fluid are less than those for Bingham fluid, power-law fluid and Herschel–Bulkley fluid. Effective viscosity increases with the yield stress, power-law index, slip and tube hematocrit but decreases with Darcy number. It is observed that the effective viscosity and mean hematocrit increase with tube radius but the core hematocrit decreases with tube radius. Further, it is noticed that the flow exhibits the anomalous Fahraeus–Lindqvist effect.

1. Introduction

Micro-circulation deals with the circulation of blood through small blood vessels such as arterioles, capillaries and venules. It consists of the complex network of blood vessels whose diameter is between 10 and 250 μm. Further, the flow of blood through smaller diameter blood vessels is accompanied by anomalous effects. One such effect is Fahraeus–Lindqvist effect, where the apparent viscosity of blood decreases with tube diameter. This effect has been confirmed by several investigators (Fahraeus and Lindqvist [1] and Dintenfass [2]).

The study of blood flow in microvessels was carried out by various authors under different assumptions (Seshadri and Jaffrin [3] and Whitmore [4]). Most of these models deal with one phase model. However, it is realized that blood being a suspension of corpuscles, behaves like a non-Newtonian fluid at lower shear rates. Haynes [5] and Bugliarello and Sevilla [6] have considered a two-fluid model with both fluids as Newtonian fluids and with different viscosities. Sharan and Popel [7] and Srivastava [8] have reported that for blood flowing through narrow tubes, there is a peripheral layer of plasma

* Corresponding author.
E-mail addresses: princenallapu@gmail.com (N. Santhosh), grk.nitw@yahoo.com (G. Radhakrishnamacharya), achamkha@pmu.edu.sa (A.J. Chamkha).
Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2015.07.009
1110-0168 © 2015 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V.

and a core region of suspension of all erythrocytes. Haldar and Andersson [9] have studied a two-layered blood flow model in which the core region is occupied by a Casson type fluid and peripheral region by Newtonian fluid. Chaturani and Upadhya [10,11] analyzed two-fluid model by assuming Newtonian fluid in peripheral region and polar fluids in core region. Chamkha et al. [12] considered a micropolar fluid flow in a vertical parallel plate channel with asymmetric heating. Philip and Chandra [13] have studied the flow of blood through uniform and stenosed tubes and analyzed the influence of slip velocity on the flow variables such as velocity, wall shear stress and flow resistance. Mohanty et al. [14] have considered the unsteady heat and mass transfer characteristics of a viscous incompressible electrically conducting micropolar fluid.

Though Newtonian and several non-Newtonian fluid models have been used to study the motion of blood, it is realized (Blair and Spanner [15]) that Herschel–Bulkley model describes the behavior of blood very closely. Herschel–Bulkley fluids are a class of non-Newtonian fluids that require a finite critical stress, known as yield stress, in order to deform. Therefore, these materials behave like rigid solids when the local shear is below the yield stress. Once the yield stress is exceeded, the material flows with a non-linear stress–strain relationship either as a shear-thickening fluid, or as a shear-thinning one. Few examples of fluids behaving in this manner include paints, cement, food products, plastics, slurries, and pharmaceutical products.

Tang and Kalyon [16] studied Herschel–Bulkley fluid flow under wall slip using a combination of capillary and squeeze flow viscometers. Huilgol and You [17] applied the augmented Lagrangian method to steady flow problems of Bingham, Casson and Herschel–Bulkley fluids in pipes of circular and square cross-sections. Maruthi Prasad and Radhakrishnamacharya [18] discussed the steady flow of Herschel-Bulkely fluid in an inclined tube of non-uniform cross-section with multiple stenoses. Taliadorou et al. [19] derived approximate semi-analytical solutions of the axisymmetric and plane Poiseuille flows of weakly compressible Herschel–Bulkley fluid with no slip at the wall. Vajravelu et al. [20] studied a mathematical model for a Herschel–Bulkley fluid flow in an elastic tube. Gorla et al. [21] investigated the combined convection from a slotted vertical plate to Micropolar fluids with slip. Rehman et al. [22] have presented the peristaltic flow and heat transfer through a symmetric channel in the presence of heat sink or source parameter. Damianou et al. [23] solved numerically the cessation of axisymmetric Poiseuille flow of a Herschel–Bulkley fluid under the assumption that slip occurs along the wall.

Recently, Santhosh and Radhakrishnamacharya [24] studied a two-fluid model for the flow of Jeffrey fluid in the presence of magnetic field through porous medium in tubes of small diameters. The objective of this paper was to study the slip effect on Herchel–Bulkley fluid flow through narrow tubes. Following the analysis of Chaturani and Upadhya [10] and Vajravelu et al. [20], the linearized equations of motion have been solved and analytical solution has been obtained. The analytical expressions for velocity, flow rate, effective viscosity, core hematocrit and mean hematocrit have been obtained. The results are depicted graphically and the effects of various relevant parameters on the flow variables have been studied.

2. Formulation of the problem

Consider the steady flow of Herschel–Bulkley fluid through a narrow tube of uniform cross-section with constant radius 'a'. It is assumed that the flow in the tube is represented by a two-layered model in which peripheral region of thickness ε ($a - b = \varepsilon$) is occupied by Newtonian fluid and the other is a central core region of radius 'b', which is occupied by Herschel–Bulkley fluid (Fig. 1). Let μ_p and μ_c be the viscosities of the fluid in peripheral region and core region, respectively. The axisymmetric cylindrical coordinate (r, z) is chosen, where r and z denote the radial and axial coordinates and the z axis is taken along the axis of the tube.

The equations governing the flow of Herschel–Bulkley fluid for the present problem (Maruthi Prasad and Radhakrishnamacharya [18] and Vajravelu et al. [20]) are given by

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) = -\frac{\partial p}{\partial z} \tag{1}$$

where τ_{rz}, the shear stress of the Herschel–Bulkley fluid, is given by

$$\tau_{rz} = \mu\left(-\frac{\partial u}{\partial r}\right)^n + \tau_0, \tau_{rz} \geqslant \tau_0 \tag{2}$$

$$\frac{\partial u}{\partial r} = 0, \quad \tau_{rz} \leqslant \tau_0 \tag{3}$$

Here u is the axial velocity, p is the pressure, τ_0 is the yield stress, μ is the fluid viscosity and $n(\geqslant 1)$ is the flow behavior index.

The region between $r = 0$ and $r = r_0$ is called plug core region and in this region, $\tau_{rz} \leqslant \tau_0$. In the region between $r = r_0$ and $r = b$, we have $\tau_{rz} \geqslant \tau_0$. Let $u = v_1(r)$ be the velocity in the peripheral region and $v_2(r)$ in the core region. Then the equations governing the flow of fluid are (Haldar and Andersson [9] and Vajravelu et al. [20]) as follows:

Peripheral region (Newtonian fluid):

$$\frac{\partial v_1}{\partial r} = -\frac{\Pr}{2\mu_p} \text{ for } b \leqslant r \leqslant a. \tag{4}$$

Core region (Herschel–Bulkley fluid):

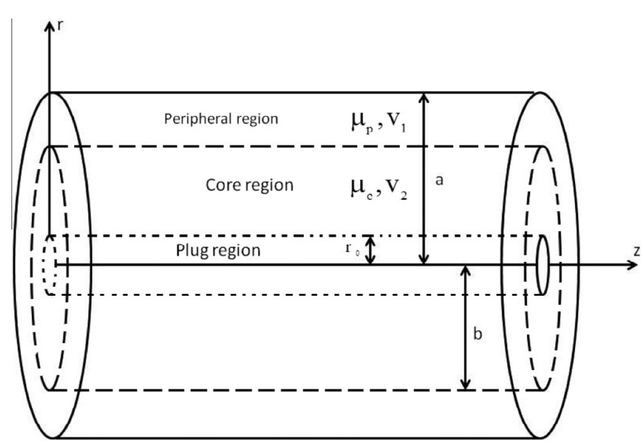

Figure 1 Geometry of the problem.