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ORIGINAL ARTICLE

Chaotic convection in a rotating fluid layer

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Abstract A study of thermal convection in a rotating fluid layer is investigated based on the dynamical systems approach. A system of differential equation like Lorenz model has been obtained by using Galerkin-truncated approximation. The chaotic convection is investigated in a rotating fluid layer. A low-dimensional, Lorenz-like model was obtained using Galerkin truncated approximation. The fourth-order Runge–Kutta method is employed to obtain the numerical solution of Lorenz-like system of equations. We found that there is proportional relation between Taylor number and the scaled Rayleigh number R. This means that chaotic behavior can be delayed (for increasing value of R) when we increase the scaled Taylor number. We conclude that the transition from steady convection to chaos depends on the level of Taylor number.

1. Introduction

Chaotic convection in fluid layer has great interest due to its relevance in a wide range of industrial applications. Chaos was obtained in a three dimensional phase space for the Lorenz [13] system arising from the truncation of the classical Rayleigh–Benard convection model. Chaotic behavior in a fluid layer can be actually advantageous in various industrial applications such as the production of crystals, oil reservoir modeling, and catalytic packed beds filtration.

The study of the effect of external rotation on thermal convection has attracted significant experimental and theoretical interest. Because of its general occurrence in geophysical and oceanic flows, it is important to understand how the Coriolis force influences the structure and transport properties of thermal convection. Rotating thermal convection also provides a system to study hydrodynamic instabilities, pattern formation and spatio-temporal chaos in nonlinear dynamical systems. The study of thermal convection in rotating fluid layer is motivated both theoretically and by its practical applications in engineering. Some of the important areas of applications in engineering include the food processing, chemical process, solidification and centrifugal casting of metals and rotating machinery. Bhadauria [6] investigated the fluid convection in a rotating porous layer under modulated temperature on the boundaries. They found that the effect of increasing the value of Taylor number is to delay the onset of convection, thus making the system more stabilizing. Similar results were found by Malashetty and Swamy [7] and Malashetty and Heera [8] for Taylor number.

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Nomenclature

Latin symbols

a	horizontal wave number
a_c	critical wave number
$\mathbf{g}$	gravitational acceleration, $(0, 0, -g)$
d	height of fluid layer
p	pressure
Pr	Prandtl number, ν/κ_T
T_a	Taylor number, $\frac{4d^4\Omega^2}{\nu^2}$
$\mathbf{q}$	velocity of the fluid (u, v, w)
Ra	Rayleigh number, $\alpha_T g d(\Delta T)d^3/\nu\kappa_T$
t	time
T	temperature
ΔT	temperature difference between the walls

Greek symbols

Ω	angular velocity, $(0, 0, \omega)$
κ_T	thermal diffusivity
α_T	thermal expansion coefficient
ρ	density
ν	kinematic viscosity, μ/ρ_0
ψ	stream function

Subscripts

b	basic state
c	critical
0	reference state
cr	critical value

Superscripts

$'$	perturbed value
$*$	non-dimensional value

There are several computational results on the effect of rotation in porous media. Vadasz and Olek [11] found that the transition from steady convection to chaos occurs by a subcritical Hopf bifurcation producing a solitary cycle which may be associated with a homoclinic explosion when the Prandtl number is low. The work of Vadasz [15] suggests an explanation for the appearance of this solitary limit cycle via local analytical results. Mahmud and Hasim [1] investigated effect of magnetic field on chaotic convection in fluid layer. He observed that transition from chaotic convection to steady convection occurs by a subcritical Hopf bifurcation producing a homoclinic explosion which may be limit cycle as Hartman number increases. Jawdat and Hasim [2] found that the onset of chaotic convection in a porous medium for a low Prandtl number, amount of internal heat generation is inversely proportional to scaled Rayleigh number. The generalized Lorenz models and their routes to chaos by energy-conserving horizontal mode truncations were investigated by Roy and Musielek [9]. They observe that $5D$ system is the lowest-order generalized Lorenz model, which can be constructed by horizontal modes. Vadasz and Olek [17] observed that when the Prandtl number is moderate, the route to chaos occurs by a period doubling sequence of bifurcations. The work of Vadasz [16] suggests an explanation for the appearance of this solitary limit cycle via local analytical results.

Mahmud and Hasim [18] investigated chaotic convection in porous media in the presence of feedback control. They observed chaotic behavior with increasing Rayleigh number. Magyari [10] demonstrated that the structure of the feedback control system proposed by Mahmud and Hasim [18] does not change the original uncontrolled system but its effect is in altering the initial conditions of the system. Sheu [4] demonstrated that interface heat transfer the route to chaos and that application of a thermal non-equilibrium model tends to stabilize steady convection. Sheu et al. [5] investigated that stress relaxation tends to accelerate the onset of chaos through the use of an oldroydian fluid. Ferrario et al. [3] studied the chaotic behavior of second grade fluid in two dimensional convection. Gupta and Singh [19] reported the effect of anisotropic parameters on chaotic convection. They found a proportional relation between scaled Rayleigh number and scaled anisotropic parameters. Gupta and Bhadauria [20] investigated the double diffusive convection in a couple stress liquid saturated porous layer with Soret effect using thermal non-equilibrium model. Gupta et al. [21] studied the effect of applied magnetic field in couple stress fluid. They found that increase in Hartmann number increases the level of chaos. Also, Gupta and Singh [22] investigated the effect of chemical reaction in double diffusive convection.

In this study, the work of Vadasz [12] on the transition to chaos in rotating porous layer is extended to include consideration of rotating fluid layer. The transition from steady convection to chaos was analyzed by using Runge–Kutta method of order four. The Galerkin truncated approximation was applied to the governing equations for thermal convection in a rotating fluid layer subject to gravity and heated from below, allowing us to deduce an autonomous system with four ordinary differential equations. This system is investigated for the dynamic behavior of thermal convection in a fluid layer and for the effect of rotation on transition to chaos.

2. Mathematical formulation

We consider a horizontal rotating fluid layer of depth d between two parallel infinite stress free boundaries, which is heated from below and cooled from above. The x-axis is taken along the lower boundary, and the z-axis vertically upward. The lower surface is held at temperature T_0, while the upper surface is at $T_0 + \Delta T$, where ΔT is temperature difference between the lower and upper surfaces. The continuity and momentum equations governing the motion of an incompressible rotating fluid are given by

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\frac{\partial \vec{q}}{\partial t} + 2\Omega \times \vec{q} = -\frac{1}{\rho_0}\nabla p + \frac{\rho}{\rho_0}\vec{g} + \nu\nabla^2\vec{q} \tag{2}$$