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ORIGINAL ARTICLE

# Minimum height of the trapezoidal filter in earth dams using Complex Function Theory

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**Abstract** Height of the trapezoidal filter in earth dam based on an impervious base is established using Complex Function Theory.

Based on different earth dam and filter dimensions, such as retained water head upstream the dam, angle of inclination and upstream face length of the filter, a new formula is derived to calculate minimum height of the trapezoidal filter.

Phreatic surface through earth dam is drawn for different dam and filter dimensions. Also, seepage discharge passing through the dam per unit length is determined.

Comparison between present work and both Hathoot and Kozeny is made.


## 1. Introduction

Complex Function Theory was used in solving problems of seepage through an earth dams such as seepage characteristics of earth dams with an "L" shaped filter by Rezk [1], and mathematical study of earth dam with upstream blanket, Rezk [2]. The problem of seepage through an earth dam with trapezoidal toe filter also was theoretically treated by Hathoot [3] using Complex Function Theory. Hathoot deduced equation of the free water surface and concluded that length of the upstream face of the filter is three times greater than that according to Casagrand [4].

Seepage through an earth dam with trapezoidal toe filter was also experimentally studied using Hele Show model to find height of the filter, Rezk [5].

Rezk and Senoon [6] established a simple formula to calculate height of the trapezoidal toe filter in earth dams.

In the present work, problem of seepage through an earth dam with trapezoidal filter and based on impervious base is mathematically treated using Complex Function Theory by considering the upstream face of the dam as a line source and the upstream face of the filter as a line sink.

Solution by Hathoot [3] for the same problem was different which considered the upstream face of the dam as a steady stream while the filter as a line sink.

Height of the trapezoidal toe filter is recommended in the present work, and the phreatic surface is compared with Hathoot [3] and Kozeny [4].

## 2. Mathematical solution

For the dam profile shown in Fig. 1, upstream face of the dam is considered as a line source and the upstream face of the filter as a line sink, applying Complex Function Theory

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**Nomenclature**

| | | | |
|---|---|---|---|
| $H$ | is the retained water head upstream the dam | $\alpha$ | is the angle of inclination of upstream face of the dam |
| $h_f$ | is the minimum height of the trapezoidal filter | $\beta$ | is the angle of inclination of upstream face of the filter |
| $K$ | is the coefficient of soil permeability | $\phi$ | is the equipotential function |
| $L$ | is the source length | $\psi$ | is the stream function |
| $m$ | is the strength of line source and line sink | | |
| $q$ | is the seepage discharge per unit length of the dam | | |
| $S$ | is the sink length | | |

to get complex potential of the system and from which equipotential and stream functions are determined [1].

*2.1. Complex potential of the line source AB*

Considering the strength of the source is $m$, the complex potential $W_1$ is determined as follows:

$$dW_1 = -\frac{m}{L}\int_0^L \ln(z - le^{i\alpha})dl = \frac{-m}{L(-e^{i\alpha})}\int_0^L \ln(z - le^{i\alpha})d(z - le^{i\alpha})$$

$$W_1 = \frac{m}{L}e^{-i\alpha}\left[(z - le^{i\alpha})\ln(z - le^{i\alpha}) - (z - le^{i\alpha})\right]_0^L = \frac{m}{L}e^{-i\alpha}\left[(z - Le^{i\alpha})\ln(z - Le^{i\alpha}) + (Le^{i\alpha}) - z\ln z\right] \quad (1)$$

Substitute $z = x + iy$, $e^{i\alpha} = cos\,\alpha + i\,sin\,\alpha$, $e^{-i\alpha} = cos\,\alpha - i\,sin\,\alpha$, and $z\ln z = z\ln r\, e^{i\theta} = (x + iy)(\ln r + i\theta)$.

$$W_1 = \varnothing_1 + i\psi_1 = \frac{m}{L}(cos\,\alpha - i\,sin\,\alpha)[x_1 lnr_1 + ix_1\theta_1 + iy_1 lnr_1 - y_1\theta_1 + L\,cos\,\alpha + iL\,sin\,\alpha - x_2 lnr_2 - ix_2\theta_2 - iy_2 lnr_2 + y_2\theta_2] \quad (2)$$

Equipotential function $\varnothing_1$ :

$$\varnothing_1 = \frac{m}{L}cos\,\alpha[x_1 lnr_1 - y_1\theta_1 + L\,cos\,\alpha - x_2 lnr_2 + y_2\theta_2] - \frac{m}{L}sin\,\alpha[-x_1\theta_1 - y_1 lnr_1 - L\,sin\,\alpha + x_2\theta_2 + y_2 lnr_2] \quad (3)$$

Stream function $\psi_1$ :

$$\psi_1 = \frac{m}{L}cos\,\alpha[x_1\theta_1 + y_1 lnr_1 + L\,sin\,\alpha - x_2\theta_2 - y_2 lnr_2] - \frac{m}{L}sin\,\alpha[x_1 lnr_1 - y_1\theta_1 + L\,cos\,\alpha - x_2 lnr_2 + y_2\theta_2] \quad (4)$$

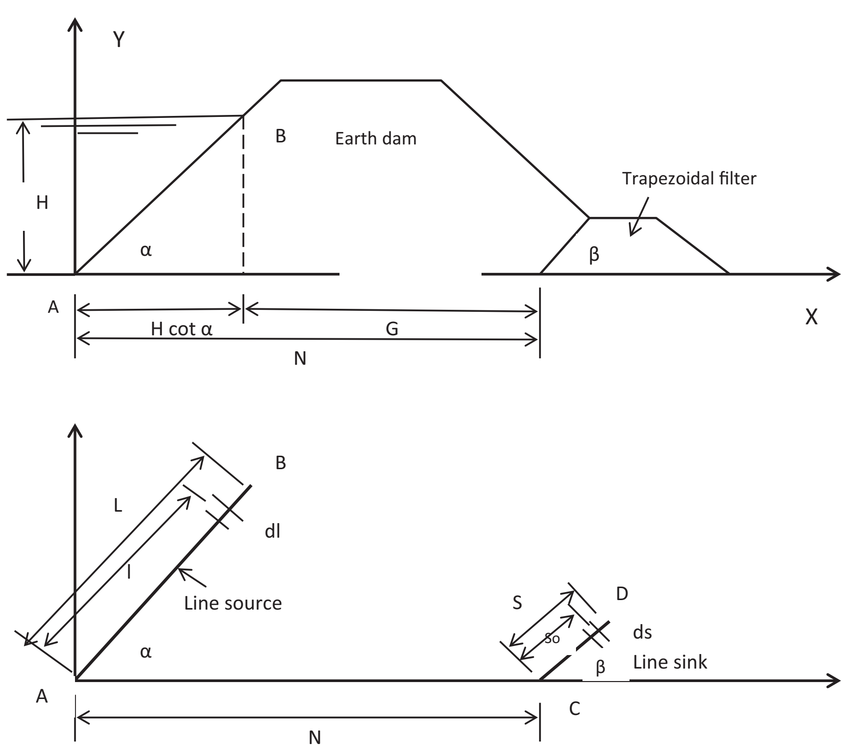


**Figure 1** The dam profile showing the line source and line sink.