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ORIGINAL ARTICLE

The unsteady flow of a third-grade fluid caused by the periodic motion of an infinite wall with transpiration

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Porous medium

Abstract In this work, the unidirectional flow of an incompressible electrically conducting third-grade fluid past a vertical transpiration wall through a porous medium with time-dependent periodic motion is presented. The nonlinear partial differential equations are transformed to ordinary differential equation by means of symmetry reductions. The reduced equation is then solved analytically for steady-state and time-dependent transient parts. The time series of the transient flow velocity for different pertinent parameters are examined through plots. During the course of computation, it was observed that the time-dependent transient and steady-state solutions agree very well at large value of time when the ratio is related to fluid parameters $\frac{\gamma_*}{\gamma} > 1$.

1. Introduction

The problems for time-dependent flow over flat walls with impulsive and periodic motions, starting with arbitrary profile, are found increased substantially during the last few years due to its several industrial applications. For a flat wall, this problem is related to the motion induced by impulsive or an oscillating semi-infinite space when the wall presents harmonic oscillations in the longitudinal direction. Interesting issues arises when taking into consideration magnetohydrodynamic flow through a porous medium. Several researchers have analyzed this for a variety of fluids in various forms. Hayat el al. [1] investigate the time-dependent magnetohydrodynamic flow of an incompressible Sisko fluid over a porous wall. The flow is due to the sudden motion of the wall plane boundary. They obtained results by invoking a symmetry approach and numerical techniques. Sharma and Khan [2] have investigated the magnetohydrodynamic flow of a viscous fluid through a porous medium induced by torsionally oscillating disk and presented approximate solutions of the flow characteristics. Ahmad and Asghar [3] gave an exact solution for magnetohydrodynamic boundary layer flow of a second grade fluid over a permeable stretching surface with arbitrary velocity and appropriate wall transpiration. Ali et al. [4] analyzed the problem of unsteady electrically conducting second grade fluid

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passing through a porous space and established an exact solutions for the transient flow due to oscillating wall boundary using Laplace transform method. Abdulhameed et al. [5] studied the unsteady magnetohydrodynamic flow of incompressible viscous fluid over flat plates with impulsive and oscillating motions, and with wall transpiration through a porous medium. They obtained results by applying an extension of the variable separation technique combined with similarity arguments.

Among important studies on third grade fluid model: Aziz and Aziz [6] examined the analytical solutions for unsteady magnetohydrodynamic flow of a third grade fluid past a porous plate within a porous medium due to an arbitrary wall with suction/injection velocity. They obtained results by applying a Lie symmetry and numerical methods. Aziz el al. [7] have presented Group invariant solutions for unsteady magnetohydrodynamic flow of a third grade fluid in porous medium due to the arbitrary velocity of non-porous plate. Other important studies related to the second-grade and third-grade fluids are [8–12].

In the studies mentioned above, the time-dependent flow of an incompressible third-grade fluid generated by an oscillating wall with transpiration in the presence of permeability of the medium under the influence of uniform magnetic field has not been discussed. Therefore, the aim of the present study is to analyze the effect of oscillating frequency, transpiration parameter, material parameters, uniform magnetic field, and porosity parameter on the flow fields in transient and steady-state flow of a third grade fluid past an infinite vertical transpiration wall through a porous medium in the presence of a uniform transverse magnetic field. To achieve this, we make use of symmetry reduction method. With this method, the governing nonlinear partial differential equation is reduced to a nonlinear ordinary differential equation, which further solved analytically for time-dependent transient and steady-state solutions.

2. Mathematical formulation

Consider the unsteady flow of a third-grade fluid occupying a porous half-space and bounded by an infinite plane wall situated in the (x, y)-plane system of Cartesian coordinate. Fig. 1 shows the physical configuration. Initially, both the plane wall and the fluid are at rest. At time $t > 0$ the wall moves in x-direction with velocity $v_w(t)$. A constant magnetic field B_0 is applied in the y-direction and there is no external electric field. The induced magnetic field is neglected under the assumption of small magnetic Reynolds number. Furthermore, symmetric nature of the flow is taken into account and pressure gradient is neglected.

The set of governing equations of an incompressible conducting third-grade fluid on porous wall embedded in a porous medium

$$\mathrm{div}(\mathbf{v}) = 0, \tag{1}$$

and the momentum equation

$$\rho \frac{d\mathbf{v}}{dt} = \mathrm{div}\mathbf{T} + \mathbf{R} - \sigma B_0^2 \mathbf{v}, \tag{2}$$

where $\mathbf{v}$ is the velocity vector, ρ is the fluid density, $\frac{d}{dt}$ is the material time derivative, $\mathbf{T}$ is the Cauchy stress tensor, $\mathbf{R}$ is the Darcy's resistance due to porous medium and σ is the electrical conductivity. The Cauchy stress tensor for a third-grade fluid is

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta_3(tr\mathbf{A}_1^2)\mathbf{A}_1, \tag{3}$$

where $\mathbf{I}$ is the identity tensor, p is the pressure, μ is the dynamic viscosity, $\alpha_1, \alpha_2, \beta_3$ are the material constants and $\mathbf{A}_i$ $(i = 1, 2, 3)$ are the Revlin–Ericksen tensors which are defined by

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T,$$
$$\mathbf{A}_n = \frac{d}{dt}\mathbf{A}_{n-1} + \mathbf{A}_{n-1}\mathbf{L} + \mathbf{L}^T\mathbf{A}_{n-1}, \quad n > 1, \tag{4}$$

where $\mathbf{L} = \nabla\mathbf{v}$.

According to Davidson [13] the magnetic Reynolds number is considered very small. It follows that the induced magnetic field produced by the fluid motion is negligible, the magnetic body force, $\mathbf{J} \times \mathbf{B}$, becomes $\sigma(\mathbf{v} \times \mathbf{B}_0) \times \mathbf{B}_0$ when imposed and induced electric fields are negligible and only the magnetic field, $\mathbf{B}_0$, contributes to the current $\mathbf{J} = \sigma(\mathbf{v} \times \mathbf{B}_0)$.

The Lorentz force on the right hand side of Eq. (2) becomes

$$\mathbf{J} \times \mathbf{B} = -\sigma B_0^2 \mathbf{v}. \tag{5}$$

where σ is the electrical conductivity.

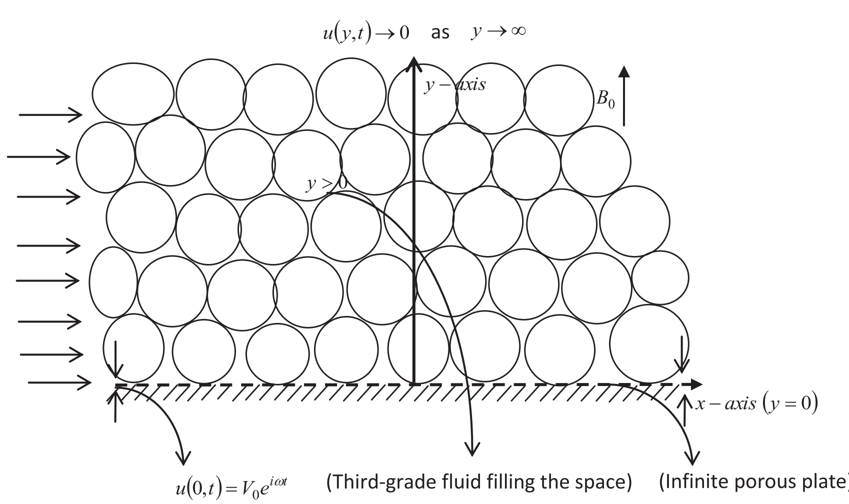

Figure 1 The physical model configuration.