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Edge states in a ferromagnetic honeycomb lattice with armchair boundaries

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ABSTRACT

We investigate the properties of magnon edge states in a ferromagnetic honeycomb lattice with armchair boundaries. In contrast with fermionic graphene, we find novel edge states due to the missing bonds along the boundary sites. After introducing an external on-site potential at the outermost sites we find that the energy spectra of the edge states are tunable. Additionally, when a non-trivial gap is induced, we find that some of the edge states are topologically protected and also tunable. Our results may explain the origin of the novel edge states recently observed in photonic lattices. We also discuss the behavior of these edge states for further experimental confirmations.

1. Introduction

One intriguing aspect of electrons moving in finite-sized honeycomb lattices is the presence of edge states, which have strong implications in the electronic properties and play an essential role in the electronic transport [1–3]. It is well known that natural graphene exhibits edge states under some particular boundaries [4,5]. For example, there are flat edge states connecting the two Dirac points in a lattice with zig-zag [1] or bearded edges [6]. On the contrary, there are no edge states in a lattice with armchair boundary [7], unless a boundary potential is applied [8].

The edge states have also been studied in magnetic insulators [9–11], where the spin moments are carried by magnons. Recently, it has been shown that the magnonic equivalence for the Kane-Mele-Haldane model is a ferromagnetic Heisenberg Hamiltonian with the Dzialozinskii-Moriya interaction [12,13]. Firstly, while the energy band structure of the magnons of ferromagnets on the honeycomb lattice closely resembles that of the fermionic graphene [14,15], it is not clear whether or not they show similar edge states, particularly in view of the interaction terms in the bosonic models which are usually ignored in graphene [16]. Secondly, most recent experiments in photonic lattices have observed novel edge states in honeycomb lattices with bearded [17] and armchair [18] boundaries, which are not present in fermionic graphene. The main purpose of this paper is to address these two issues. By considering a ferromagnetic honeycomb lattice with armchair boundaries, we find that the bosonic nature of the Hamiltonian reveals novel edge states which are not present in their fermionic counterpart. After introducing an external on-site potential at the outermost sites, we find that the edge states are tunable. Interestingly, we find that the nature of such edge states is Tamm-like [19], in contrast with the equivalent model for armchair graphene [8] but, as mentioned earlier, in agreement with the experiments in photonic lattices [17,18]. Furthermore, after introducing a Dzialozinskii-Moriya interaction (DMI), we find that the topologically protected edge states are sensitive to the presence of the Tamm-like states and they also become tunable.

2. Model Hamiltonian

We consider the following Hamiltonian for a ferromagnetic honeycomb lattice,

$$H = -J\sum_{\langle i,j\rangle} \mathrm{S}_i \cdot \mathrm{S}_j + \sum_{\langle\langle i,j\rangle\rangle} D_{ij} \cdot \left(\mathrm{S}_i \times \mathrm{S}_j\right), \tag{1}$$

where the first summation runs over the nearest-neighbors (NN) and the second over the next-nearest-neighbors (NNN), $J > 0$ is the isotropic ferromagnetic coupling, S_i is the spin moment at site i and $\mathbf{D}_{ij}$ is the DMI vector between NNN sites [20]. If we assume a lattice in the x-y plane, according to Moriya's rules [20], the DMI vector vanishes for the NN but has non-zero component along the z direction for the NNN. Hence, we can assume $\boldsymbol{D}_{ij} = D\nu_{ij}\hat{\boldsymbol{z}}$, where $\nu_{ij} = \pm 1$ is an orientation dependent coefficient in analogy with the Kane-Mele model [21]. For the infinite system in the linear spin-wave approximation (LSWA), the Hamiltonian in Eq. (1) can be reduced to a bosonic equivalent of the Kane-Mele-Haldane model [12–14]. To investigate the edge states we consider an armchair boundary along the x direction, with a large N sites in the y direction, as shown in Fig. (1). A partial Fourier transform is made and

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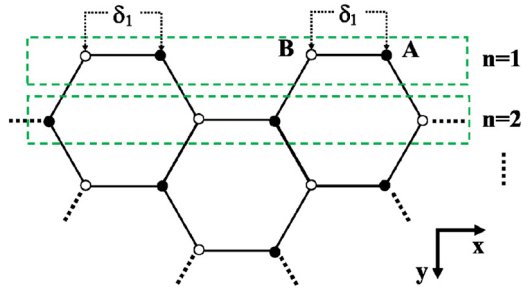

Fig. 1. Squematics of the upper armchair edge of a honeycomb lattice. The external on-site potential δ_1 is applied at the outermost sites. Here, n is a real-space row index in y direction perpendicular to the edge. For a large N, we consider the opposite edge with the same structure and with an on-site potential δ_N.

the Hamiltonian given in Eq. (1) in LSWA can be written in the form,

$$H = -t\sum_k \Psi_k^\dagger M \Psi_k, \tag{2}$$

where $\Psi_k^\dagger = \left[\Psi_{k,A}^\dagger, \Psi_{k,B}^\dagger\right]$ is a $2N\times 2N$, 2-component spinor, k is the Bloch wave number in the x direction and $t = JS$. The matrix elements of M are $N\times N$ matrices given by,

$$\begin{aligned} M_{11} &= \left(1-\delta_1\right)T^\dagger T + \left(1-\delta_N\right)TT^\dagger + \delta_s I + M_D,\\ M_{12} &= -J_1 I - J_2\left(T+T^\dagger\right),\\ M_{21} &= M_{12}^\dagger,\\ M_{22} &= M_{11} - 2M_D, \end{aligned} \tag{3}$$

with $\delta_s = \left(1+\delta_1+\delta_N\right)I$ and $M_D = J_3\left(TT - T^\dagger T^\dagger\right) + J_4\left(T^\dagger - T\right)$ the DMI contribution. Here, T is a displacement matrix as defined in Ref. [22] and I a $N\times N$ identity matrix. We have also introduced two on-site energies δ_1 and δ_N at the outermost sites of each boundary, respectively. The coupling terms are: $J_1 = e^{-ik}$, $J_2 = e^{ik/2}$, $J_3 = iD'$, $J_4 = 2iD'\cos\left(3k/2\right)$ and $D' = D/J$. The numerical diagonalization of the matrix given by Eq. (2) reveals that the bulk spectra is gapless only if $N = 3m+1$, with m a positive integer [23]. However, to avoid size-dependent bulk gaps or hybridization between edge states of opposite edges [8], we consider a large N where the edge states are independent of the size [24,25].

3. Edge states and boundary conditions

From the explicit form of the matrix elements given in Eq. (3), the coupled Harper equations can be obtained [26]. If we assume that the edge states are exponentially decaying from the armchair boundary, we can consider the following anzats [27,28] for the eigenstates of M in Eq. (2),

$$\Psi_k(n) = \begin{bmatrix}\psi_{k,A}(n)\\ \psi_{k,B}(n)\end{bmatrix} = z^n\begin{bmatrix}\phi_{k,A}\\ \phi_{k,B}\end{bmatrix}, \tag{4}$$

where $\left[\phi_{k,A}, \phi_{k,B}\right]^t$ is an eigenvector of M, z is a complex number and $n\{=1,2,3,\ldots\}$ is a real space lattice index in the y direction, as shown in Fig. (1). Upon substitution of the anzats in the coupled Harper equations, the complex number z obey the following polynomial equation,

$$\sum_{\mu=0}^{4} a_\mu (z+z^{-1})^\mu = 0, \tag{5}$$

with coefficients: $a_0 = 1-(3-\varepsilon)^2 - 4J_4^2$, $a_1 = 8J_3J_4 + J_1^*J_2 + J_2^*J_1$, $a_2 = -4J_3^2 + J_4^2 + 1$, $a_3 = -2J_3J_4$ and $a_4 = J_3^2$. For a given k and energy ε, such a polynomial always yields four solutions for $(z+z^{-1})$. Since we require a decaying wave from the boundary, only the solutions with $|z|<1$ are relevant for the description of the edge states at the upper edge and $|z|>1$ for the lower (opposite) edge. The eigenfunction of Eq. (2) satisfying $\lim_{n\to\infty}\Psi_k(n) = 0$ may now in general be written as,

$$\psi_{k,l}(n) = \sum_{\upsilon=1}^{4} c_\nu z_\nu^n \phi_{l,\nu}, \tag{6}$$

where the coefficients c_υ are determined by the boundary conditions and $\phi_{l,\nu}$ is the two-component eigenvector $(l = A,B)$ of M. From the Harper equations provided by the Eq. (3) and Eq. (4), the boundary conditions are satisfied by,

$$\left(1-\delta_1\right)\psi_{k,A}(1) - J_2\psi_{k,B}(0) = 0, \tag{7}$$

$$\left(1-\delta_1\right)\psi_{k,B}(1) - J_2^*\psi_{k,A}(0) = 0, \tag{8}$$

$$J_4\psi_{k,A}(0) - J_3\psi_{k,A}(-1) = 0, \tag{9}$$

$$J_4\psi_{k,B}(0) - J_3\psi_{k,B}(-1) = 0. \tag{10}$$

By Eq. (6), the above relations can be written as a set of equations for the unknown coefficients c_ν. The non-trivial solution and the polynomial given by Eq. (5), provide us a complete set of equations for the edge state energy dispersion and they can be solved numerically. The same procedure can be followed to obtain the solutions for the opposite edge.

4. Results and discussions

4.1. Zero DMI

For the system without DMI, the coupling terms involving J_3 and J_4 vanish, and the boundary conditions are reduced to the Eqs. (7) and (8) with a quadratic polynomial in $(z+z^{-1})$ of Eq. (5). In particular, for the (uniform) case with $\delta_1 = \delta_N = 1$, the edge and the bulk sites have the same on-site potential and the boundary conditions provide us with two bulk solutions with $z^2 = 1$. Therefore, in analogy with graphene with armchair edges, there are not edge states [7]. However, as shown in Fig. (2a), in the absence of external on-site potential $\left(\delta_1 = \delta_N = 0\right)$, two new dispersive localized modes are obtained. Located between (red, continuous line) and below (green, dotted) the bulk bands, such edge states are well defined along the Brillouin zone and their energy bands are doubly degenerated due to the fact that there are two edges in the ribbon. These edge states have not been previously predicted or observed in magnetic insulators. However, we believe that they are analogous to the novel edge states recently observed in a photonic honeycomb lattice with armchair edges [18]. Although in Ref. [18] these edge states may be attributed to the dangling bonds along the boundary sites (the details have been given for zig-zag and bearded but not for armchair edges), and since these dangling bonds can be viewed as effective defects along the edges, similar physics is contained in our model where the effective defects are described by the different on-site potential at the boundaries. We believe that our approach has the advantage of simple implementation for various boundary conditions. In particular, we have obtained expressions for the wavefunctions and their confinement along the boundary. The latter is given by the penetration length (or width) of the edge state [29] defined as,

$$\xi_i(k) \equiv \frac{\sqrt{3}}{2}\left[\ln\left|\frac{1}{z_i(k)}\right|\right]^{-1}, \tag{11}$$

indicating a decay of the form $\sim e^{-y/\xi_i(k)}$. In the above equation, z_i is the i-th decaying factor in the linear combination, Eq. (6). Since we require two decaying factors to construct the edge state, we have two penetration lengths as mentioned in Ref. [18]. The penetration lengths for the edge states with $\delta_1 = \delta_N = 0$ are shown in the Fig. (2b). The