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Asymmetric acoustic transmission in graded beam

Li Jing [a], Jiu Hui Wu [a,*], Dong Guan [a], Kuan Lu [a], Nansha Gao [b], Cao Songhua [a]

[a] School of Mechanical Engineering and State Key laboratory for Strength and Vibration of Mechanical Structures, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, China
[b] School of Marine Science and Technology, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China

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ABSTRACT

We demonstrate the dynamic effective material parameters and vibration performance of a graded beam. The structure of the beam was composed of several unit cells with different fill factors. The dispersion relations and energy band structures of each unit cell were calculated using the finite element method (FEM). The dynamic effective material parameters in each unit cell of the graded beam were determined by the dispersion relations and energy band structures. Longitudinal wave propagation was investigated using a numerical method and FEM. The results show that the graded beam allows asymmetric acoustic transmission over a wide range of frequencies.

1. Introduction

In the last few decades, phononic crystals have attracted much attention owing to their unique acoustic properties [1–9]. In particular, their effective material parameters can be negative [1,2], anisotropic [3], or dependent on the frequencies of vibration [4,5], resulting in some unusual phenomena, such as low-frequency forbidden bands [5,6] and negative refraction [7]. These exotic effects have been intensely studied and many intriguing devices have been designed, such as the invisible cloak [3], superlens [7], and acoustic shield [8]. With recent developments in graded phononic crystals, the innovative properties of this structure promise a wide variety of applications, such as a plane lens and acoustic absorber. By introducing the concept of a gradient-index, Lin et al. [9] designed a graded phononic crystal to control the propagation of acoustic waves. The gradient refractive index profile in a gradient-index phononic crystal is obtained by adjusting the material parameters [9], fill factor [9,10] and lattice constants [11]. Researchers have developed graded phononic crystals and investigated the focusing of the Lamb waves [12] and Rayleigh waves [13], both numerically and experimentally. Using graded phononic crystals, Yu [14] designed a broadband acoustic absorber.

In parallel, by using a nonlinear material [15,16], an asymmetric structure [17,18], and a graded structure [19–23], the asymmetric acoustic transmission effect has been realized. Asymmetric acoustic transmission is important in many applications, such as acoustic rectifying devices or sound diodes. Researchers have proposed two-dimensional prism models consisting of phononic crystals [19,20] or acoustic metamaterials [21] for unidirectional transmission. Prism structures can be regarded as spatial gradient structures. Chen et al. [22] proposed graded grating phononic crystal slabs that support broad bi-directional asymmetric Lamb wave transmission. Zhu et al. [23] designed a four-body composite structure, which consists of two reflectors and two metasurfaces. The metasurfaces are fabricated by using graded grooves. By changing the thickness of a graded plate, Krylov et al. [24,25] investigated the "acoustic black holes" for flexural waves. However, most of the aforementioned works focus on the properties of wave propagation in graded phononic crystals or acoustic metamaterials. Besides, it is necessary to investigate the relationship between the properties of wave propagation and effective material parameters of graded phononic crystals.

In this work, a gradient fill factor of each unit cell is introduced into the phononic crystals to obtain graded beam. The dispersion relations and energy band structures of each unit cell are calculated using the finite element method (FEM). Then, the dynamic effective material parameters in each unit cell of the graded beam are determined using the dispersion relations and energy band structures. The vibration performance of the graded beam is then investigated by using a numerical method and FEM. The paper is organized as follow. In Section 2, the model of the structure is presented and explained. And the calculation methods of dispersion relations and energy band structures based on FEM are described. The setup of numerical model of effective materials parameters is presented. In Section 3, the vibration performance of

* Corresponding author.
E-mail addresses: lj94172350@hotmail.com (L. Jing), ejhwu@mail.xjtu.edu.cn (J.H. Wu).

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the graded beam is investigated based on the FEM and numerical method. Finally, conclusions are given in Section 4.

2. Effecive parameters of a graded beam

The model studied is shown in Fig. 1. A graded beam is composed of several unit cells with length a and radius r. Each unit cell is composed of beam-A (white regions in Fig. 1) with length $a_1=(1-F)a$ and beam-B (gray regions in Fig. 1) with length $a_2=Fa$. Beam-A and beam-B are made of polymethylmethacrylate (PMMA) and steel, respectively. The material properties of PMMA are $\rho_{PMMA}=1142\ \text{kg}\cdot\text{m}^{-3}$, $E_{PMMA}=2$ GPa, and $\sigma_{PMMA}=0.389$; and those for steel are $\rho_{steel}=7782\ \text{kg m}^{-3}$, $E_{steel}=210.6$ GPa, and $\sigma_{steel}=0.3$. By adjusting the fill factor F of beam-B, the gradient material properties of the beam can be changed.

To obtain the effective material parameters of the graded beam, each unit cell is treated as a cell of a periodic beam. Thus, the effective density and elastic modulus of each unit cell can be calculated by using its the energy band structure [26]. The dispersion relation and energy band structure of each unit cell are calculated by using the FEM. The periodic boundary conditions are considered on two sides of a unit cell. The dispersion relations and eigenmodes are obtained by varying the Bloch wave vector in the first Brillouin zone and by solving a spectral problem. The eigenvectors represent the modal displacement fields. The mechanical energy can be calculated from the deformation of the structure.

Fig. 2(a) shows the dispersion relations for a longitudinal wave in eleven periodic beams with the various fill factors. As the fill factor increases, the dispersion bands of the periodic beams first fall and then rise along the ΓX orientation. We found that the fill factor of the lowest dispersion curve is equal to 0.5. In fact, when the fill factor tends to 0.5, the proportion of two materials in the periodic beam will approach the same value, and this change causes a stronger dispersion. Conversely, when the fill factor tends to 0 or 1, the dispersion curves will rise and approach the dispersion curve of the beam of the dominant material. Fig. 2 (b) shows the relationships between the wave vector and mechanical energy of each unit cell with various fill factors. It should be noted that the energy is not only related to the wave vector, but also to the vibration amplitude. Thus, the vibration amplitude should be normalized to eliminate the effect of the amplitude on the energy band structures. The proportion of steel increases with the fill factor. In addition, the mechanical energy density of steel is higher than that of PMMA when their vibration amplitudes are equal. Thus, the energy bands of the periodic beams rise along the ΓX orientation as the fill factor increases.

We consider each unit beam to be effectively equivalent to a homogeneous beam with effective density, ρ_{eff}, and elastic modulus, E_{eff}. This homogeneous beam has the same size as the unit beam, as shown in Fig. 3. Meanwhile, the dispersion relations and energy band structures of the homogeneous beam are the same as those of the unit beam.

For the longitudinal waves propagating in the homogeneous beam, the equation of motion is given by

$$E_{eff}S\frac{\partial^2 u}{\partial x^2}+\rho_{eff}S\frac{\partial^2 u}{\partial t^2}=0, \tag{1}$$

where u denotes the displacement of a structure and S is the cross-sectional area. The harmonic solution of Eq. (1) is

$$u(x,t)=A\sin(kx-\omega t), \tag{2}$$

where A denotes the vibration amplitude of a structure and k and ω are the wave vector and angular frequency, respectively. Substituting Eq. (2) into Eq. (1), the dispersion relation can be expressed as

$$\rho_{eff}\omega^2=E_{eff}k^2 \tag{3}$$

The effective density and elastic modulus cannot be uniquely determined by dispersion relations, as shown in Eq. (3). According to the Ref. [27], the accompanying energy equation of Eq. (1) is easily obtained

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_{eff}S\left(\frac{\partial u}{\partial t}\right)^2+\frac{1}{2}E_{eff}S\left(\frac{\partial u}{\partial x}\right)^2\right)+\frac{\partial}{\partial x}\left(-E_{eff}S\frac{\partial u}{\partial x}\frac{\partial u}{\partial t}\right)=0 \tag{4}$$

The first term, which takes derivative with respect to t, represents the energy density and the second term, which takes derivative with respect to x, represents the energy flux. Substituting Eq. (2) into the energy density term in Eq. (4), the energy density, w, is approximately given by

$$w=\frac{1}{2}\rho_{eff}S\omega^2A^2\sin^2(kx-\omega t)+\frac{1}{2}E_{eff}Sk^2A^2\cos^2(kx-\omega t) \tag{5}$$

Considering the average values of $\cos^2(kx\text{-}\omega t)$ and $\sin^2(kx\text{-}\omega t)$ in one period are both equal to one half, we have

$$w=\frac{1}{4}\rho_{eff}S\omega^2A^2+\frac{1}{4}E_{eff}Sk^2A^2 \tag{6}$$

Eq. (6) shows that the energy at each point is equal to the sum of the kinetic and potential energy at that point. The total energy, W, of the longitudinal vibration of the beam is calculated using

$$W=\int_0^a w\,dx \tag{7}$$

In a mechanical structure without damping, the maximum kinetic energy, T_{max}, the maximum potential energy, U_{max}, and the total energy are equal:

$$W=T_{max}=U_{max} \tag{8}$$

Substituting Eq. (3) and Eq. (7) into Eq. (8), we obtain the maximum kinetic and potential energies of a homogeneous beam.

$$\begin{cases} T_{max}=W=\frac{1}{2}\rho_{eff}S\omega^2A^2a \\ U_{max}=W=\frac{1}{2}E_{eff}Sk^2A^2a \end{cases} \tag{9}$$

Eq. (9) shows that the maximum kinetic and potential energies are functions where the dependent variables are the effective material parameters, ρ_{eff} and E_{eff}. Thus, the effective material parameters can be expressed as

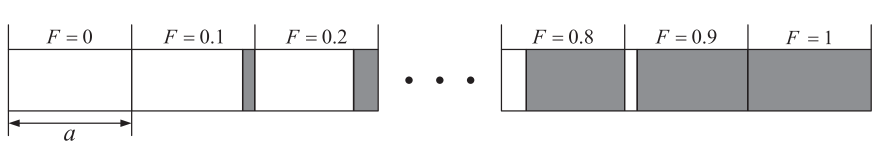

Fig. 1. Configuration of the graded beam.