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# Scaling of quantum Fisher information close to the quantum phase transition in the *XY* spin chain

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ABSTRACT

The quantum phase transition of an *XY* spin chain is investigated by employing the quantum Fisher information encoded in the ground state. It is shown that the quantum Fisher information is an effective tool for characterizing the quantum criticality. The quantum Fisher information, its first and second derivatives versus the transverse field display the phenomena of sudden transition, sudden jump and divergence, respectively. Besides, the analysis of finite size scaling for the second derivative of quantum Fisher information is performed.



## 1. Introduction

Quantum phase transition (QPT) in the ground state of a quantum many-body system arises purely from quantum fluctuation and gives rise to qualitatively different phases by varying an external parameter [1]. QPT can be detected at very low temperatures, where the thermal fluctuation is not strong enough to cover the quantum fluctuation. In the critical regime, the energy-level crossing between the ground and first excited states usually emerges, which leads to the dramatic change of the ground-state wave function as well as the divergence of the correlation length. How to precisely reveal the phenomenon of QPT in a many-body system is crucial. Traditionally, the occurrences of non-vanishing order parameters or symmetry breaking are adopted as the indicators of QPT. Alternative tools have been put forward for characterizing QPT from the viewpoint of quantum information science, such as quantum correlation [2–10], geometric phase [11–13], quantum fidelity [14] and trace distance [15,16]. The key issue is that the nonclassical information embedded in a quantum state shall be sensitive to the variation of quantum fluctuation. Besides, according to quantum mechanics, quantum fluctuation may also cause uncertainty of measurements on observables. Then, quantum Fisher information (QFI), a typical measure for estimation precision of an unknown parameter [17–19], may be naturally related to QPT in several many-body systems [20–22]. Moreover, QFI is proposed as a measure of the statistical distinguishability on the space of the density operators [23] and the non-Markovianity in open quantum systems [24], which is also closely related to the fidelity susceptibility [25,26]. A superiority of the QFI in comparison with other parameters is that the QFI close to a QPT may not be influenced by weak symmetry-preserving perturbations [27]. However, the experimental extraction of QFI is still a problem of fundamental interest in quantum metrology. Recently, certain progresses have been made in this aspect, such as Refs. [27,28]. In Ref. [27], the measurement of QFI via the Kubo response function has been demonstrated in quantum Ising models. In Ref. [28], an alternative scheme of extracting QFI by means of Bayesian analysis has also been reported.

On the other hand, quantum spin systems, a class of important models describing magnetism of materials in condensed matter physics, have served as a natural platform for exploring QPTs. This benefits from the fact that several quantum critical behaviors can be presented in different quantum spin models and many of these models can be simulated by cold atomic gases in optical lattices [29]. Recently, the QPTs of quantum spin systems have been intensively studied from the viewpoint of quantum information science [9]. For instance, the pairwise entanglement between two nearest-neighbor spins is extremely sensitive to the QPT parameter at the critical point for a critical *XY* spin chain [2,3], which leads to a logarithmic divergence of its first derivative. By contrast, for another kind of nonclassical correlation, i.e., quantum discord,

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the first derivative merely shows an inflexion point and its second derivative instead shows a quadratic logarithmic divergence [8]. It is then interesting to explore the approach of detecting quantum criticality of an *XY* spin chain with QFI.

It has been reported in Refs. [25,26] that the fidelity susceptibility of the phase-transition parameter is sensitive to critical phenomena. In this paper, we alternatively consider the anisotropic spin-1/2 *XY* chain in a transverse magnetic field with QFI of a phase parameter adiabatically encoded in the ground state. The difference is that we do not focus directly on the QFI of the transverse magnetic field and explore certain new phenomena for the QFI near the critical point. It is shown that the QFI, its first and second derivatives versus the strength of the external field display the phenomena of sudden transition, sudden jump and divergence, respectively, which are rather different from those of nonclassical correlations such as quantum entanglement and quantum discord. Besides, we discuss the finite-size scaling behavior of the second derivative of QFI. Our results suggest that the QFI is indeed an effective and convenient tool for detecting QPT.

This paper is organized as follows. In Section 2, we first outline the basic concepts of QFI. In Section 3, we introduce the system considered as the anisotropic spin-1/2 *XY* chain in a transverse magnetic field and explore the QFI encoded in the ground state of the *XY* model. Section 4 is devoted to the discussion of the finite scaling behavior of the QFI. Conclusions are presented in Section 5.

## 2. Quantum Fisher information

First, we briefly outline the basic concept of QFI. For a parameterized density matrix operator $\varrho_\vartheta$ with $\vartheta$ being the parameter to be estimated, the QFI is defined by [17–19]

$$\mathcal{F}_\vartheta = \mathrm{Tr}(\varrho_\vartheta \mathcal{L}^2), \tag{1}$$

where $\mathcal{L}$ is the symmetric logarithmic derivative operator given by

$$\partial_\vartheta \varrho_\vartheta = \frac{1}{2}(\mathcal{L}\varrho_\vartheta + \varrho_\vartheta \mathcal{L}). \tag{2}$$

By diagonalizing the density matrix $\varrho_\vartheta$ as $\varrho_\vartheta = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$ with $p_i \geq 0$, the QFI can be expressed as [30]

$$\mathcal{F}_\vartheta = \sum_i \frac{(\partial_\vartheta p_i)^2}{p_i} + 2\sum_{i\neq j} \frac{(p_i - p_j)^2}{p_i + p_j}\left|\langle\Psi_i|\frac{\partial}{\partial\vartheta}|\Psi_j\rangle\right|^2. \tag{3}$$

When $\varrho_\vartheta$ is a pure state, Eq. (3) reduces to

$$\mathcal{F}_\vartheta = 4(\langle\partial_\vartheta\Psi|\partial_\vartheta\Psi\rangle - |\langle\Psi|\partial_\vartheta\Psi\rangle|^2). \tag{4}$$

An important feature of the QFI is related to the achievable lower bound of uncertainty about the parameter $\vartheta$ via the quantum Cramér–Rao theorem [17–19]

$$\mathrm{Var}(\vartheta) \geq \frac{1}{M\mathcal{F}_\vartheta}, \tag{5}$$

where $\mathrm{Var}(\vartheta)$ is the mean square error in the parameter $\vartheta$ and $M$ represents the number of independent measurements.

## 3. Detection of quantum phase transition with quantum Fisher information

The system under consideration is a spin-1/2 ferromagnetic chain with nearest exchange coupling $J$ in a transverse magnetic field of strength $\lambda$. The Hamiltonian is given by [31]

$$H = -\sum_{j=1}^{N}\left\{\frac{J}{2}[(1+\gamma)\sigma_j^x\sigma_{j+1}^x + (1-\gamma)\sigma_j^y\sigma_{j+1}^y] + \lambda\sigma_j^z\right\}, \tag{6}$$

where $0 \leq \gamma \leq 1$ is the anisotropy parameter, $N$ is the number of spins, and $\sigma_j^{x,y,z}$ are the standard Pauli operators at the $j$th site with cyclic boundary conditions ($\vec{\sigma_j} = \vec{\sigma}_{j+N}$) assumed. For $0 < \gamma \leq 1$, the model belongs to the Ising universality class. It reduces to the transverse *XX* model and the transverse Ising model for the two particular cases of $\gamma = 0$ and $\gamma = 1$, respectively. The system undergoes a second-order phase transition from the ferromagnetic phase to the paramagnetic phase at the quantum critical point $J/\lambda = 1$ [31]. This model is extensively employed to discuss the properties of one-dimensional and quasi-one-dimensional compounds such as $Cs_2CoCl_4$ and $PrCl_3$ [32,33].

It has been reported that the fidelity susceptibility (mathematically equivalent to QFI) of the phase-transition parameter is sensitive to critical phenomena [25,26]. To explore the quantum criticality of the *XY* model, we alternatively use an adiabatic variation of the Hamiltonian which can be realized by applying a rotation of $\phi$ around the $z$ direction to each spin [11,12]. The difference is that we do not focus directly on the QFI of the phase-transition parameter and the parameter $\phi$ to be estimated is a phase introduced adiabatically. The parameterized Hamiltonian can be expressed as

$$H(\phi) = O(\phi)HO^\dagger(\phi); \quad O(\phi) = \prod_{j=1}^{N} e^{i\phi\sigma_j^z/2}. \tag{7}$$

It is worth noting that the critical behavior of the *XY* spin chain is independent of parameter $\phi$ due to the fact that the energy spectrum of this system is $\phi$ independent [11,12]. This parameterized Hamiltonian can be exactly diagonalized by the standard technique of Jordan–Wigner and Bogoluibov transformations with the ground state of $H(\phi)$ derived as [11,12]

$$|G\rangle_\phi = \prod_{k>0}\left(\cos\frac{\theta_k}{2}|0\rangle_k|0\rangle_{-k} - ie^{2i\phi}\sin\frac{\theta_k}{2}|1\rangle_k|1\rangle_{-k}\right), \tag{8}$$

where $|0\rangle_k$ and $|1\rangle_k$ represent the vacuum state and excitation state in the $k$th fermionic mode with $k = 2\ell\pi/N$ and $\ell = 1, 2, \ldots, (N-1)/2$ for odd $N$. Here, the parameter $\theta_k$ is defined by $\theta_k = \arctan[\gamma\sin k/(\cos k - \lambda)]$ where we have set $J = 1$. According to Eq. (4), the QFI in the ground state of *XY* spin chain reads as

$$\mathcal{F}_\phi = \frac{4}{N}\sum_{k>0}\sin^2\theta_k, \tag{9}$$

where we have introduced a factor $1/N$ for normalization.

In the thermodynamic limit $N \to \infty$, the discrete summation in Eq. (9) is replaced by the continuous integral [34,35]

$$\frac{2}{N}\sum_{k>0} \to \frac{1}{\pi}\int_0^\pi \mathrm{d}k. \tag{10}$$

Then the QFI in Eq. (9) can be rewritten as

$$\mathcal{F}_\phi = \frac{2}{\pi}\int_0^\pi \frac{\gamma^2\sin^2 k}{\gamma^2\sin^2 k + (\cos k - \lambda)^2}\mathrm{d}k, \tag{11}$$

which can be analytically calculated for the Ising spin-chain case ($\gamma = 1$) by using the following integral formula

$$\int_0^\pi \frac{\sin^2 x}{p + q\cos x}\mathrm{d}x = \frac{p\pi}{q^2}\left(1 - \sqrt{1 - \frac{q^2}{p^2}}\right). \tag{12}$$

As a consequence, we can obtain a concise expression for the QFI in the Ising chain as