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ORIGINAL ARTICLE

Delayed system control in presence of actuator saturation

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Abstract The paper is introducing a new design method for systems' controllers with input delay and actuator saturations and focuses on how to force the system output to track a reference input not necessarily saturation-compatible. We propose a new norm based on the way we quantify tracking performance as a function of saturation errors found using the same norm. The newly defined norm is related to signal average power making possible to account for most common reference signals e.g. step, periodic. It is formally shown that, whatever the reference shape and amplitude, the achievable tracking quality is determined by a well defined reference tracking mismatch error. This latter depends on the reference rate and its compatibility with the actuator saturation constraint. In fact, asymptotic output-reference tracking is achieved in the presence of constraint-compatible step-like references.

1. Introduction

Controlling linear systems with input saturation has been much studied especially over the last two decades, see e.g. [1] and references list therein. The solutions proposed so far have been developed following two main paths, called, respectively, anti-windup compensator (AWC) synthesis and direct control design (DCD). The first approach consists of designing a controller that ensures satisfactory control performances in the absence of actuator saturation case. Then, a static or dynamic compensator is designed to minimize the effect of actuator saturation on the closed-loop performances. In the DCD method, the input constraint is taken into account at the controller design phase. In addition to actuator saturation, physical systems are also subject to (less or more significant) dead-times [2].

The conjunction of these two ubiquitous factors, if it is not appropriately accounted for in the control design stage, may cause drastic deterioration of control performances. The point is that relatively few works have dealt with the problem of controlling delayed systems with saturating actuators. In this respect, a number of global bounded stabilization [3–7] and asymptotic local stabilization results [8–10] have been reached using direct control designs. Stabilization results have also been achieved using anti-windup based designs [11,12]. For instance, in [12] the AWC is designed to ensure L_2-stability

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of the operator relating the control saturation error $\tilde{u} = u - sat(u)$ (difference between the control input signal generated with and without input saturation) to $\tilde{z} = z - \bar{z}$ (error between the control system performance outputs with and without input saturation).

This L_2-stability result is certainly interesting due to its generality. But, the class of admissible inputs (references, disturbances) for which the condition $\tilde{u} \in L_2$ holds is not explicitly defined. Furthermore, it is not clear how the constrained closed-loop system behaves when the inputs are not constraint-compatible so that $\tilde{u} \notin L_2$.

In this note, the focus is made on global asymptotic tracking of arbitrary-shape reference signals for systems with constrained input with delay. We will show that asymptotic reference tracking is achievable for constraint-compatible step-like reference signals. To this end, a saturating controller is developed within the ring of pseudo-polynomial using the (finite-spectrum) pole-placement technique [13,14].

The point is that, in practical situations, reference compatibility may be difficult to check or even lost due to model uncertainties. Then, it is of practical interest to analyze the tracking capability of the proposed controller facing constraint-incompatible reference signals of arbitrary shape. This issue has never been investigated in the context of input-constrained dead-time systems. It is presently dealt with, for the considered class of saturated controllers, making use of available input-output L_2-stability tools [15]. The novelty is that the obtained tracking performance is assessed using a new, more suitable, norm representing signal average power (rather than energy). The new norm induces a normed space, denoted L_{2a}, that contains all bounded signals (while the energy-related L_2-space does not). Then, L_{2a} turns out to be a quite suitable framework to address the tracking issue in the presence of arbitrary-shape (possibly not constraint-compatible) references. Making use of the L_{2a}-norm, it is formally shown that the proposed saturated controller features quite interesting output-reference matching properties. Accordingly, the tracking accuracy is related to a reference tracking mismatch error, depending on reference rate and its constraint-compatibility error. The smaller the tracking mismatch error, the better the average tracking quality. This holds independently of the input shape and amplitude.

The present paper is an improved and more complete version of the conference paper [17]. It is organized as follows: Section 2 is devoted to formulating the control problem; the controller is designed in Section 3 and analyzed in Section 4; the corresponding tracking performances are illustrated by simulation in Section 5.

2. Control problem statement

We are interested in controlling input-delayed linear systems of the form:

$$A(s)\hat{y}(s) = B(s)e^{-s\tau}\hat{u}(s) \tag{1}$$

with:

$$\begin{aligned} A(s) &= s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 \\ B(s) &= b_{n-1}s^{n-1} + \cdots + b_1 s + b_0 \end{aligned} \tag{2}$$

in the presence of the input constraint:

$$|u(t)| \leqslant u_M \tag{3}$$

where $\hat{u}(s)$ and $\hat{y}(s)$ are the Laplace transforms of $u(t)$ and $y(t)$, the system input and output (respectively); u_M denotes the maximal allowed amplitude of the control signal; the integer n and the real numbers (a_i, b_i) are the system order and parameters, respectively. It is supposed that $A(s)$ is Hurwitz polynomial and $(sA(s), B(s))$ are coprime. These assumptions guarantee system controllability even in the presence of the input limitation (3). The only assumption on $B(s)$ is that $b_0 \neq 0$ so that the system static gain is nonzero. That is, $B(s)$ may be Hurwitz or not, allowing the system (1) to be nonminimum phase [16]. The aim of the study was to develop a controller that makes the tracking error,

$$e_y = y - y^* \tag{4}$$

as small as possible, whatever the initial conditions, where y^* denotes an arbitrary-shape bounded reference signal. The point is that the system nonminimum phase nature makes perfect tracking (e.g. $e_y \in L_2$) unachievable (even in the unconstrained case) in the presence of arbitrary-shape reference signals. In fact, perfect tracking is only achievable (in the unconstrained case) for reference signals generated by a model of the form $D(s)\hat{y}^*(s) = 0$ with $D(s)$ is any polynomial with simple zeros on the imaginary axis. Then, it is well known that perfect tracking can be achieved by incorporating $D(s)$ in the control law, in accordance with the internal model principle [16]. Presently, we make the common choice $D(s) = s$ which leads to control laws with integral action, allowing for perfect tracking of constant references (as $D(s)\hat{y}^*(s) = 0$ is then equivalent to $\dot{y}^*(t) = 0$).

In turn, the input constraint (3) introduces a structural limitation of the class of references that can be perfectly matched. Specifically, perfect matching cannot be achieved if the reference y^* is not constraint-compatible. In the case of constant references, constraint-compatibility is simply characterized by the condition $|u^*(t)| \leqslant u_M$ with:

$$u^*(t) = \frac{A(0)}{B(0)} y^*(t+\tau) = \frac{a_0}{b_0} y^*(t+\tau)$$

where $A(0)/B(0)$ is nothing other than the inverse of the system static gain. An equivalent formulation of reference constraint-compatibility is that $u^* - sat(u^*) = 0$ where $sat(\cdot)$ denotes the function of saturation defined by:

$$sat(z) = \min(u_M, |z|)\ sgn(z), \quad z \in \boldsymbol{R} \tag{5}$$

Now, the controller we seek must be able to guarantee perfect asymptotic tracking in the presence of constant constraint-compatible references. Otherwise, the tracking quality must depend on how much the reference is deviating from the ideal shape defined by the equations $\dot{y}^*(t) = 0$ and $u^* - sat(u^*) = 0$. The instantaneous deviation is conveniently represented by the output-reference mismatch error $|\dot{y}^*| + |u^* - sat(u^*)|$. The smaller this error is the better must be the tracking quality. This objective is presently formalized requiring that the performance operator,

$$|\dot{y}^*| + |u^* - sat(u^*)| \to |e_y| \tag{6a}$$

is L_2 stable. Accordingly [e.g. 15], there exists a pair of positive real constants (α, β) such that one has, for all bounded input y^* and any real $T > 0$: