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Scaling rules for critical current density in anisotropic biaxial superconductors

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ABSTRACT

Recent researches highlight the additional anisotropic crystallographic axis within the superconducting plane of high temperature superconductors (HTS), demonstrating the superconducting anisotropy of HTS is better understood in the biaxial frame than the previous uniaxial coordinates within the superconducting layer. To quantitatively evaluate the anisotropy of flux pinning and critical current density in HTS, we extend the scaling rule for single-vortex collective pinning in uniaxial superconductors to account for flux-bundle collective pinning in biaxial superconductors. The scaling results show that in a system of random uncorrected point defects, the field dependence of the critical current density is described by a unified function with the scaled magnetic field of the isotropic superconductor. The obtained angular dependence of the critical current density depicts the main features of experimental observations, considering possible corrections due to the strong-pinning interaction.

1. Introduction

High-temperature superconductors (HTS) show an extensive application prospect in large-power magnets and cables [1,2], due to its relatively high critical temperature and critical current density. How to improve the vortex-pinning properties for raising the critical current density and upper critical field, is a huge challenge in the area of high power application. An outstanding feature of HTS is the anisotropy of superconducting condensate [3–5], which manifests at the inequality of the microscopic superconductivity parameters or the upper critical field [6] along the crystallographic axes. For example, in single-crystal $NdFeAsO_{1-x}F_x$, the ratio of zero-temperature coherence lengths perpendicular to the FeAs layers and in the layers is ~ 4 [7]. In $(Ba,K)Fe_2As_2$ and Nd (F,O)FeAs the anisotropy factors are ~ 2.5 and 7.5 [8], respectively.

The field and angular dependences of the critical current density J_c shed light on the anisotropy. Multiple techniques have been used to measure J_c variation with the magnetic field direction in various superconducting samples [9–14]. The critical current density is fundamentally caused by the interaction of the flux vortices and defects. [15] To this end, the anisotropic J_c depends not only on the anisotropy of superconducting condensate itself but on the defective nature. In high-temperature cuprate superconductors, columnar and planar defects exhibit a notably different field dependence of J_c [13,16–18]. Furthermore, the anisotropy in J_c depends on the strength of the pinning interaction. From the observations on the behaviors of J_c anisotropy, a strong-pinning interaction is demonstrated in the hole-doped $Ba_{0.6}K_{0.4}Fe_2As_2$ single crystal [14], whereas there is a weak-collective-pinning interaction after introducing point pinning defects. In coated conductors containing $BaZrO_3$ (BZO) nanorod, the lattice mismatch between BZO and the matrix induces the weak uncorrelated pinning interaction (point pinning defects) [19]. This pinning raises the critical current density for the whole field-angle range, broadens the *ab*-plane peak range and reduces the critical current density anisotropy. Xu et al. [20] investigate the field and temperature dependences of the critical current density in two types of $YBa_2Cu_3O_x$ thin films with different pinning landscapes, demonstrating that the insulating precipitates with strain mismatch (point defect pinning) render a significantly reduced effective Ginzburg–Landau (G–L) anisotropy parameter and a relatively high bulk flux pinning force density.

A traditional way to incorporate the anisotropy into the phenomenological description of superconductivity is to introduce an anisotropic effective-mass tensor into the Ginzburg–Landau (GL) equations [21]. One then has to repeat all the calculations that have been done for the isotropic case before. A more elegant

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approach is the scaling rule [21,22], which scales the anisotropic problem to a corresponding isotropic one at the initial level of GL free energy. Reusing the scaling rule, the isotropic results are then simply generalized to the anisotropic ones. Despite its effectiveness, the traditional scaling rules are limited to treat the anisotropic uniaxial superconductors. On the other hand, single-vortex pinning receives the most concerns in the scaling rules, without considering the magnetic field dependence. Here, inspired by most recent findings on the biaxial anisotropy [23–27] (axes a, b and c) and on the field dependence of the critical current density [13,28–33], we extend the scaling rules to account for the anisotropic biaxial superconductors exposed to magnetic field with arbitrary direction and magnitude. Through this paper, a simple theory is established to unfold the complex physics in the anisotropy of the most concerned HTS. In the next section we first give the procedure for deducing the scaling rules for anisotropic biaxial superconductors within the single-vortex pinning regime, in light of those for uniaxial superconductor [21].

2. Scaling rules for anisotropic biaxial superconductors

2.1. Single-vortex pinning regime

The GL free-energy functional for the Gibbs free energy per unit volume is [21,34]

$$f_s = f_n(0) + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \sum_{j=a,b,c}\frac{1}{2m_j}\left|\left(-i\hbar\frac{d}{dx_j} - 2eA_j\right)\Psi\right|^2 + \frac{B^2}{2\mu_0} - \mathbf{B}\cdot\mathbf{H} \quad (1)$$

where $\Psi(\mathbf{r})$ is the order parameter, A_j is the magnetic vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$ is the local magnetic induction strength, and $\mathbf{H}$ is the magnetic field. $f_n(0)$ is the free energy of the normal state at zero magnetic field, $B^2/2\mu_0$ is magnetic field energy and $(-\mathbf{B}\cdot\mathbf{H})$ is the diamagnetic energy. The phenomenological GL parameter $\alpha(T) = -\alpha(0)(1 - T/T_c)$ changes sign at the critical temperature T_c, whereas β is taken to be a positive constant with respect to temperature. $e(>0)$ is the elementary charge. $m_j (j = a, b, c)$ denote the effective masses along the principal axes of the crystal.

For simplicity and because HTS are within high accuracy biaxial (axes $\|c$, $\|a$ and $\|b$) materials, we denote the mass anisotropy ratio by $\varepsilon^2 = m_{ab}/m_c$ and $\zeta^2 = m_a/m_b$ in which $m_{ab} = \sqrt{m_a m_b}$. In addition, we define $\lambda_{ab} = \sqrt{\lambda_a \lambda_b}$ and $\xi_{ab} = \sqrt{\xi_a \xi_b}$. The definitions are appropriate for anisotropic biaxial superconductors [24]. Here, the superconducting anisotropy comes from the difference of effective masses along crystal axes, as defined in the anisotropic GL theory [21]. In this sense, the anisotropy ratio of the effective mass is equivalent to the anisotropy ratio of the coherence length or London penetration depth as introduced in Refs. [24,30]. However, the anisotropic GL theory and the "mass anisotropy law" break down in the weakly coupled two-band superconductors such as MgB_2 [35]. The magnetic field $\mathbf{H}$ encloses an angle θ with the z axis, and its projection in the xy plane encloses an angle φ with the x axis (see Fig. 1(a)).

The anisotropy enters in the GL free energy (1) only through the gauge-invariant gradient term, which becomes isotropic if we choose the following scales of the coordinate axes and vector potential,

$$x = \zeta^{-1/2}\tilde{x},\ y = \zeta^{1/2}\tilde{y},\ z = \varepsilon\tilde{z},\ A_x = \zeta^{1/2}\widetilde{A_x},\ A_y = \zeta^{-1/2}\widetilde{A_y},\ A_z = \varepsilon^{-1}\widetilde{A_z} \quad (2)$$

where we denote a quantity q in the scaled isotropic system by $\tilde{q}$.

The magnetic flux density $\mathbf{B} = \nabla \times \mathbf{A}$ is then scaled as:

$$B_x = \zeta^{-1/2}\varepsilon^{-1}\widetilde{B_x},\ B_y = \zeta^{1/2}\varepsilon^{-1}\widetilde{B_y},\ B_z = \widetilde{B_z} \quad (3)$$

Applying Eq. (3) in the free energy expression (1), one finds that the last two terms representing the magnetic energy are transformed into

$$f_m = (2\mu_0)^{-1}(\zeta^{-1}\varepsilon^{-2}\widetilde{B_x}^2 + \zeta\varepsilon^{-2}\widetilde{B_y}^2 + \widetilde{B_z}^2) - (\zeta^{-1/2}\varepsilon^{-1}\widetilde{B_x}H_x + \zeta^{1/2}\varepsilon^{-1}\widetilde{B_y}H_y + \widetilde{B_z}H_z) \quad (4)$$

Note that, the anisotropy is reintroduced in f_m, although it vanishes in the gradient term. In general, it is not possible to render both terms in the Gibbs energy isotropic simultaneously. If the superconductor is strongly type II (GL parameter $\kappa >> 1$) or if the magnetic field are large enough [21], the magnetic field is nearly uniform on the elementary length scales, and we can adopt a mean-field decoupling scheme, in which we first minimize the magnetic-field energy f_m with respect to $\tilde{\mathbf{B}}$ and then insert the resulting uniform field back into the free energy. Minimizing f_m (Eq. (4)) with respect to $\widetilde{B_x}$, $\widetilde{B_y}$ and $\widetilde{B_z}$, the applied external magnetic field is scaled in the isotropic system as,

$$H_x = \mu_0^{-1}\zeta^{-1/2}\varepsilon^{-1}\widetilde{B_x},\ H_y = \mu_0^{-1}\zeta^{1/2}\varepsilon^{-1}\widetilde{B_y},\ H_z = \mu_0^{-1}\widetilde{B_z} \quad (5)$$

Combing this result with Eq. (3), we find the constitutive relation $\mathbf{B} = \mu_0\mathbf{H}$ in the anisotropic system. Thus, with the aid of $B_x = B\sin\theta\cos\varphi$, $B_y = B\sin\theta\sin\varphi$ and $B_z = B\cos\theta$, in the rescaled isotropic system the magnetic field is related to the original magnetic field as

$$\widetilde{B} = \varepsilon_{\theta\varphi}B, \quad (6)$$

where $\varepsilon_{\theta\varphi}{}^2 = \varepsilon^2(\theta,\varphi) = \varepsilon^2\sin^2\theta(\zeta\cos^2\varphi + \zeta^{-1}\sin^2\varphi) + \cos^2\theta$. In uniaxial superconductors with $\zeta = 1$, $\varepsilon_{\theta\varphi}{}^2$ is reduced to $\varepsilon_\theta{}^2 = \varepsilon^2\sin^2\theta + \cos^2\theta$ and thus $\widetilde{B} = \varepsilon_\theta B$, which coincides with the appropriate result in Ref. [21]. In Fig. 2, we plot the angular dependence of B, indicating that the degree of freedom increases to two in biaxial superconductors, and varying the two anisotropy parameters renders nonlinear and gradual changes in the profiles of $B(\theta,\varphi)$.

Consider a "longitudinal vector" $\mathbf{l}_l = l_l(\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta)$ directed along the magnetic field $\mathbf{H}$ (see Fig. 3). Using Eq. (2) we obtain

$$\tilde{\mathbf{l}}_l = (\tilde{l}_x, \tilde{l}_y, \tilde{l}_z) = l_l(\zeta^{1/2}\sin\theta\cos\varphi, \zeta^{-1/2}\sin\theta\sin\varphi, \varepsilon^{-1}\cos\theta),$$
$$l_l = \varepsilon\varepsilon_{\theta\varphi}{}^{-1}\tilde{l}_l,$$
$$\mathbf{l}_l \| \mathbf{H}. \quad (7)$$

Consider a "transverse vector" $\mathbf{l}_t = (l_{tx}, l_{ty}, l_{tz})$ with $\hat{l}_{tx} = \cos\theta\cos\varphi\cos\psi_t - \sin\varphi\sin\psi_t$, $\hat{l}_{ty} = \cos\theta\sin\varphi\cos\psi_t + \cos\varphi\sin\psi_t$ and $\hat{l}_{tz} = -\sin\theta\cos\psi_t$, which lies in the plane perpendicular to the vortex line. The direction of $\mathbf{l}_t$ in this plane is defined by an angle ψ_t (Fig. 3). We have the following scaling rules for $\mathbf{l}_t$,

$$\tilde{\mathbf{l}}_t = l_t(\zeta^{1/2}\hat{l}_{tx}, \zeta^{-1/2}\hat{l}_{ty}, \varepsilon^{-1}\hat{l}_{tz}),$$
$$\tilde{l}_t{}^2 = l_t{}^2[\zeta(\cos\theta\cos\varphi\cos\psi_t - \sin\varphi\sin\psi_t)^2 + \zeta^{-1}(\cos\theta\sin\varphi\cos\psi_t + \cos\varphi\sin\psi_t)^2 + \varepsilon^{-2}\sin^2\theta\cos^2\psi_t],$$
$$\mathbf{l}_t \perp \mathbf{B}. \quad (8)$$

If we denote the radius of the vortex core by r_c, the scaled $\tilde{r}_c$ in the isotropic system is then $\tilde{\mathbf{r}}_c = r_c(\zeta^{1/2}\hat{l}_{tx}, \zeta^{-1/2}\hat{l}_{ty}, \varepsilon^{-1}\hat{l}_{tz})$. Since $\tilde{\mathbf{r}}_c$ is not perpendicular to the vortex line direction $\tilde{\mathbf{l}}_l$, ξ_{ab} is given by the projection $r_{c\perp}$ of $\tilde{\mathbf{r}}_c$ on the plane perpendicular to $\tilde{\mathbf{l}}_l$,